1.2 : Number Systems and Conversions

Decimal (base 10)	Binary (base 2)	octal (base 8)	Hexadecimal (base 16)
0 - 9	0, 1	0 - 7	0 - F
	8 4 2 1		
00	0000	00	0
01	0001	01	1
02	0010	02	2
03	0011	03	3
04	0100	04	4
05	0101	05	5
06	0110	06	6
07	0111	07	7
08	1000	10	8
09	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

$$(10)_{10} = (1010)_2 = (12)_8 = (A)_{16}$$

Convert from any base to Decimal (Base 10)

any Base —(by using Positional Notation)→ Decimal

- Binary to Decimal

3 2 1 0 -1 -2 -3

$$(1001.101)_2 = 2^3 + 2^0 + 2^{-1} + 2^{-3}$$
$$= 8 + 1 + \frac{1}{2} + \frac{1}{8}$$
$$= (9.625)_{10}$$

- octal to Decimal

1 0 -1 -2

$$(75.34)_8 = (7 \times 8^1) + (5 \times 8^0) + (3 \times 8^{-1}) + (4 \times 8^{-2})$$
$$= 56 + 5 + 0.375 + 0.0625$$
$$= (61.4375)_{10}$$

- Hexadecimal to Decimal

1 0 -1 -2

$$(A2.13)_{16} =$$

Note

A	B	C	D	E	F
10	11	12	13	14	15

$= (10 \times 16^{1}) + (2 \times 16^{0}) + (1 \times 16^{-1}) + (3 \times 16^{-2})$

$= 160 + 2 + \frac{1}{16} + \frac{3}{256} = 162.0742188$

- Base 5 to Decimal

base 5 = 0, 1, 2, 3, 4

$(323)_5 = (3 \times 5^{2}) + (2 \times 5^{1}) + (3 \times 5^{0})$ (2 1 0)

$= 75 + 10 + 3 = (88)_{10}$

Convert from Decimal to any base

Decimal	by using Division method →	any Base

- Decimal to Binary (Divide by 2)

$(21)_{10} = (10101)_2$

		Result	Remainder
21	2	10	1
10	2	5	0
5	2	2	1
2	2	1	0
1	2	0	1

direction ↑

Another method :

2^4	2^3	2^2	2^1	2^0
16	8	4	2	1

$(21)_{10} = (1 \quad 0 \quad 1 \quad 0 \quad 1)_2$

$16 + 4 + 1 = 21$

- Decimal to octal

$(61)_{10} = (75)_8$

61 | 8
56
5 | 7 | 8
7

- Divide by 8
- Keep track of the reminder

- Decimal to Hexadecimal

$(590)_{10} = (24E)_{16}$

590 | 16
-576
14 | 36 | 16
-32
4 | 2 | 16
2

direction

- Divide by 16
- Keep track of the reminder

Decimal with floating point (binary with fraction)

$(0.3)_{10} = (0.01001)_2$

$0.3 \times 2 = 0.6$ → $0.6 \times 2 = 1.2$ → $0.2 \times 2 = 0.4$ → $0.4 \times 2 = 0.8$ → $0.8 \times 2 = 1.6$

take only the decimal point for the next step

$(0.625)_{10} = (101)_2$

$0.625 \times 2 = 1.25$ → $0.25 \times 2 = 0.5$ → $0.5 \times 2 = 1.0$

Exercise:

① $(13.125)_{10} = (\quad)_2$

② $(5.963)_{10} = (\quad)_{16}$

Conversion from any base to another

First convert to decimal (base-10) then convert to the target base.

Convert from base-7 to Hexadecimal

(1) Convert from base-7 to decimal

(2) Convert from decimal to Hexadecimal

$(642)_7 = (144)_{16}$

(1) $(642)_7 = (6 \times 7^2) + (4 \times 7^1) + (2 \times 7^0)$

$= 294 + 28 + 2 = (324)_{10}$

(2) $(324)_{10} = (144)_{16}$

324	16
20	16
1	16

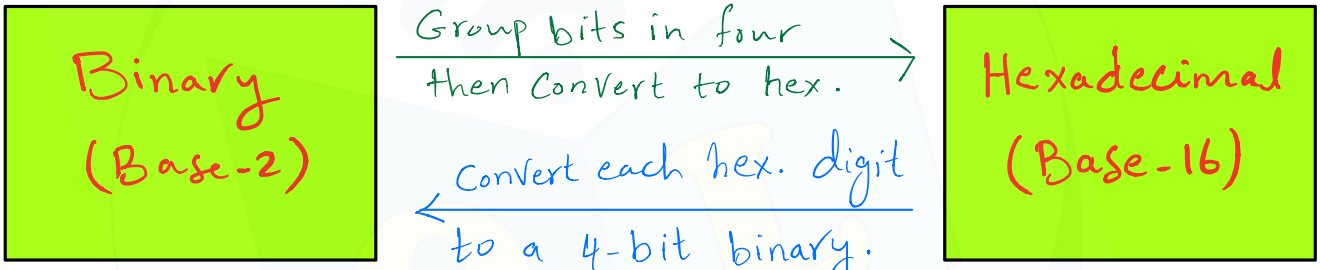
Result | Remainder

20	4
1	4
0	1

Special Case Conversion (bases with Power of 2)

base 2	base 4	base 8	base 16
2^1	2^2	2^3	2^4
1-bit	2-bit	3-bit	4-bit

- Binary to Hexadecimal and vice verse



$(1001101.010111)_2 = (\quad)_{16}$

$(0100\,1101.0101\,1100)_2 = (4D.5C)_{16}$

4 — add 0's to the left

D — start

start — 5

C — add 0's to the right

$(ABC7)_{16} = (1010101111000111)_2$

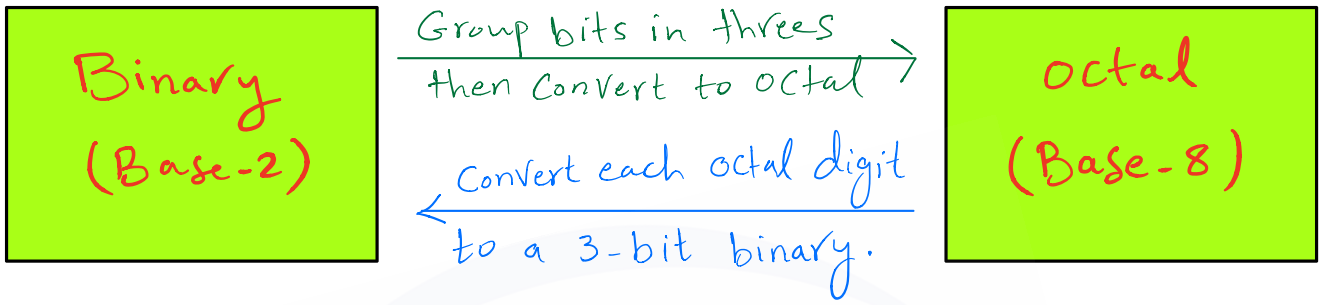
1010 — A

1011 — B

1100 — C

0111 — 7

- octal to Binary and vice verse



$(735)_8 = (111011101)_2$

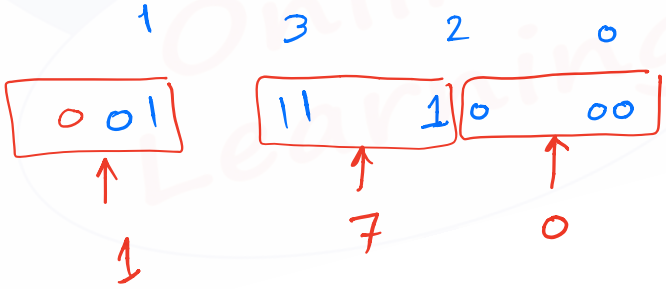
7	3	5
111	011	101

2-bit 3-bit

- Base-4 to Base-8

Convert each base-4 digit to a 2 bit binary then Group them in threes bit.

$(1320)_4 = (170)_8$



3-bit 4-bit

- octal to Hexadecimal

(use binary as an intermediary)

$(5272)_8 = (\quad)_{16}$

1). Convert each octal digit to a 3-bit binary.

5	2	7	2
101	010	111	010
↗ MSB			↑ LSB

2) Group bits in fours, then convert to hex. Start from LSB.

LSB : Least Significant bit
MSB : Most Significant bit

1010	1011	1010
A	B	A

$(5272)_8 = (ABA)_{16}$

Exercise:

1. $(10010001)_2 = (\quad)_8$

2. $(999)_{10} = (\quad)_9$