ITCS255
Midterm Exam Revision

1. Please choose the best correct answer for each of the following questions.

   1. Suppose thatf(x) is an upper bound function ong(x). Which of the following iscorrect?
      a. $g(x) \in O(f(x))$
      b. $g(x) \in \Omega(f(x))$
      c. $g(x) \in \Theta(f(x))$

   2. Suppose a and b are integers and $a|b$. Which of the following is correct?
      a. $a|(b-5)$
      b. $a|5b$
      c. $b|a$
      d. $\gcd(a, b) = 1$

   3. The linear congruence $a\,x \equiv b \pmod{m}$ has a unique solution if
      a. $\gcd(a, b) = 1$
      b. $\gcd(b, m) = 1$
      c. $\gcd(a, m) = 1$
      d. $\gcd(a, m)|b$

   4. The linear combination of gcd(17,13) is
      a. $17\times(-2) + 13\times5$
      b. $17\times(-1) + 13\times5$
      c. $17\times(-3) + 13\times4$
      d. $17\times(-2) + 13\times4$

5. Which of the following is true?
   a. 15≡4 (mod 7)
   b. 15≡2 (mod 3)
   c. 21≡3 (mod 6)
   d. 17≡4 (mod 6

6. For the functions, $n^{1000000}$ and 2^n, what is the asymptotic relationship between these func-tions?
   a. $n^{1000000}$ is $O(2^n)$
   b. $n^{1000000}$ is $\Omega(2^n)$
   c. $n^{1000000}$ is $\Theta(2^n)$

7. Given that 8≡3 (mod 5) and 9≡4 (mod 5), which of the following is true ?
   a. $8^9 \equiv 3^4$ (mod 5)
   b. 9/8 ≡ 4/3 (mod 5)
   c. 8 + 9 ≡ 3 + 4 (mod 5)
   d. 8 ≡ 4 (mod 5) and 9 ≡ 3 (mod 5)

2. Use Arithmetic Modularoperations to find 15^{5630}(mod 14)

3. Let n and m be integers. Prove that if n | m, then n | $(4n - 5m)^2$

4. Use the Basic Definition of big-Oto prove that n + log_2(n+ 1)∈O(n). Apply the Ad-hoc calculations method.

5. Use infinite limits to prove that $\frac{4n^3+2}{n^2}$ ∈Θ(n)

6. Consider the linear congruence $55x \equiv 1570 \pmod{22570}$
   a. How many solutions does it have? Simplify it so that it has a unique solution

   b. Find the unique solution to the simplified linear congruence you found in part(a). Show your steps.

7. Let $f(n) = n^2 - 4n + 1$. Use the basic definition to prove that $f(n) \in O(n^2)$.

8. If $k \in \mathbb{N}$, prove that $\gcd(3k+2, 5k+3) = 1$.

9. Suppose a+b ≡ 4 (mod 5) and 3a+b ≡ 12 (mod 5). Find the value of a

10. Solve the linear congruence 19x≡1 (mod 77)

11. Give a big-Oestimate forf(n) = (n lgn+n^2)(n^3+ 2)

12. UseForward Chainingto solve the following recurrence relation.

$$a_0 = 0, \quad a_0 = a_{n-1} + 3n, \quad n \geq 1$$

13.Use Back Substitution method to solve the recurrence relation

$$a_1 = 3, \; a_n = 7a_{n-1} + 6, \quad n \geq 2$$