MATHS 101 – Calculus I

Final Exam Revision

Question 1:

(20 points) Let $f(x) = x^3 - 3x^2 + 4$.

a) Find the intervals of increase and decrease.
b) Find the local maximum value(s) and local minimum value(s).
c) Find the intervals of concavity.
d) Find the inflection point(s).

Question 2:

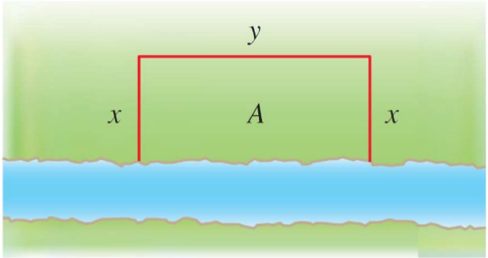
(20 points) Evaluate the following integrals. Show your work for full credit.

$$\int_{-2}^{4} (|x| - 3)\, dx$$

$$\int \left(3^x - x^{-1} + \frac{1}{x^2 + 1}\right)\, dx$$

Question 3:

(12 points) A farmer has 1200 m of fencing and wants to fence off a rectangular field that borders a straight river. He needs no fence along the river. What are the dimensions of the field that has the largest area?



Question 4:

(12 points) Use logarithmic differentiation to find the derivative of

$$y = \frac{x^{\sin x} e^{2x+3}}{\cos^2 x}$$

Question 5:

(16 points) Choose the correct answer in each of the following:

(1) $\lim_{x \to 0} \frac{|x|}{x} =$

(A) 1

(B) -1

(C) 0

(D) ∞

(E) $-\infty$

(F) DNE

(2) If $g(x) = \int_{x^2+1}^{x^4+x^2+1} \ln(t)\ dt$, then $g'(x) =$

(A) $(4x^3 + 2x)\ln(x^4 + x^2 + 1) + 2x\ln(x^2 + 1)$

(B) $(x^4 + x^2 + 1)\ln(x^4 + x^2 + 1) + (x^2 + 1)\ln(x^2 + 1)$

(C) $(4x^3 + 2x)\ln(x^4 + x^2 + 1) - 2x\ln(x^2 + 1)$

(D) $(x^4 + x^2 + 1)\ln(x^4 + x^2 + 1) - (x^2 + 1)\ln(x^2 + 1)$

(E) $\frac{4x^3+2x}{x^4+x^2+1} + \frac{2x}{x^2+1}$

(F) $\frac{4x^3+2x}{x^4+x^2+1} - \frac{2x}{x^2+1}$

(3) The derivative of $y = \ln\sqrt{x}$ is

(A) $y' = \frac{\sqrt{x^3}}{2}$

(B) $y' = \frac{1}{\sqrt{x}}$

(C) $y' = \frac{1}{2\sqrt{x}}$

(D) $y' = -\frac{1}{2x}$

(E) $y' = -\frac{1}{\sqrt{x}}$

(F) $y' = \frac{1}{2x}$

(4) If $y = \frac{x^3}{e^x}$, then $y' =$

(A) $\frac{3x^2 - x^3}{e^{4x}}$

(B) $\frac{3x^2 - x^3}{2e^x}$

(C) $\frac{3x^2 + x^3}{e^{2x}}$

(D) $\frac{3x^2 - x^3}{e^{2x}}$

(E) $\frac{3x^2 + x^3}{e^x}$

(F) $\frac{3x^2 - x^3}{e^x}$

(5) The value(s) of c that satisfy the conclusion of the mean value theorem for $f(x) = \frac{1}{x}$ on $[1,3]$ is

(A) $c = \sqrt{3}$

(B) $c = \pm\sqrt{3}$

(C) $c = \frac{\sqrt{3}}{2}$

(D) $c = 1$

(E) $c = 0$

(F) No such c exists

(6) Write $5\sinh x + 7\cosh x$ in terms of e^x and e^{-x}

(A) $12e^x - 12e^{-x}$

(B) $12e^x + 2e^{-x}$

(C) $6e^x + e^{-x}$

(D) $5e^x + 7e^{-x}$

(E) $6e^x - e^{-x}$

(F) $2e^x - 12e^{-x}$

(7) The linearization of the function $f(x) = e^{3x}$ at $a = 0$ is $L(x) =$

(A) $1 + x$

(B) $1 - x$

(C) $1 + 3x$

(D) $3x$

(E) $1 - 3x$

(F) None of the above

(8) $\int \frac{x^4+\sqrt[3]{x}}{x^2}\ dx =$

(A) $\frac{x^3}{3} - \frac{3}{2\sqrt[3]{x^2}} + C$

(B) $3x^3 - \frac{2}{3}\sqrt[3]{x^2} + C$

(C) $\frac{\frac{1}{5}x^5 + \frac{4}{3}x^{\frac{4}{3}}}{\frac{1}{3}x^3} + C$

(D) $x^3 + \sqrt[3]{x^2} + C$

(E) $\frac{x^3}{3} + \frac{2}{3}\ x^{-\frac{2}{3}} + C$

(F) None of the above

(9) **BOUNS** The value(s) of x at which the function $f(x) = e^{2x} - 18x$ has a horizontal tangent line

(A) $x = \frac{e^9}{2}$

(B) $x = \frac{2}{\ln 9}$

(C) $x = -\frac{\ln 9}{2}$

(D) $x = \ln 4.5$

(E) $x = \frac{\ln 9}{2}$

(F) None of the above

(10) **BOUNS** The equation of the tangent line of the function $f(x) = e^{2x} - 18x$ at the point $(0,1)$ is

(A) $y = 1 + 16x$

(B) $y = 1 - 16x$

(C) $y = 1 - 18x$

(D) $y = 1 + 18x$

(E) $y = 2 - 16x$

(F) None of the above

(11) **BOUNS** If $r(x) = f\left(g\big(h(x)\big)\right)$ and $h(1) = 5$, $g(5) = 3$, $h'(1) = 4$, $g'(5) = 10$ and $f'(3) = 8$, then $r'(1) =$

(A) 320

(B) 80

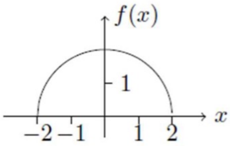
(C) 32

(D) 400

(E) 200

(F) None of the above

(12) **BOUNS** The following is the graph of $f(x) = \sqrt{4 - x^2}$. Determine $\int_0^2 f(x)\, dx$



(A) $\tan^{-1}(2) - \tan^{-1}(0)$

(B) $\frac{\pi}{4}$

(C) π^2

(D) π

(E) $\cot^{-1}(2) - \cot^{-1}(0)$

(F) $\csc^{-1}(2) - \csc^{-1}(0)$

3.9:Related Rates:

1) If $\sqrt{x} + \sqrt{y} = 5$ when $x = 4, y = 9$ and $\frac{dx}{dt} = 8$ then $\frac{dy}{dt} =$

3) If $\int_0^1 f(x)\,dx = -2$ and $\int_0^5 f(x)\,dx = 3$ then $\int_1^5 f(x)\,dx =$

9) Given that $\frac{dv}{dt} = \frac{8}{1+t^2} + \sec^2 t$ and V(0)=1. Find $V(t)$