Multiple Choice Questions.

1. $\int x(2x+5)^8\,dx =$
   A. $\frac{1}{40}(2x+5)^{10} - \frac{5}{36}(2x+5)^9 + C$
   B. $\frac{1}{10}(2x+5)^{10} - \frac{5}{9}(2x+5)^9 + C$
   C. $\frac{5}{9}(2x+5)^9 + C$
   D. $\frac{1}{40}(2x+5)^{10} - \frac{1}{2}x^2 + C$
   E. 0
   F. $\frac{1}{2}x^2(2x+5)^9 + C$

2. $\lim_{x\to 0^+}(\cos x)^{\frac{1}{x^2}}$
   A. $e^{-\frac{1}{2}}$
   B. 0
   C. $-\frac{1}{2}$
   D. e
   E. $e^{\frac{1}{2}}$
   F. 1

3. The series $\frac{1}{(1+5x)^2}$ is represented as a power series as
   A. $\sum_{n=0}^{\infty}(-1)^{n+1}5^{n+1}(n+1)x^n$
   B. $\sum_{n=0}^{\infty}(-1)^n 5^n n x^n$
   C. $\sum_{n=1}^{\infty}(-1)^n 5^n n x^n$
   D. $\sum_{n=0}^{\infty}(-1)^n 5^n (n+1)x^{n+1}$
   E. $\sum_{n=0}^{\infty}(-1)^n 5^n (n+1)x^n$
   F. $\sum_{n=1}^{\infty}(-1)^n 5^n (n+1)x^{n-1}$

4. **The coefficient of x^4 in the Maclaurin series of $(e^x - 1)\, sin\, x$ is**
   - A. $-\frac{1}{3!}$
   - B. $\frac{1}{4!}$
   - C. $\frac{1}{3!}$
   - D. -1
   - E. 1
   - F. 0

5. **Consider the convergent alternating series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^8}$. What is the least number of terms of the series we need to add in order to ensure that the error in approximating the sum satisfies $|\text{error}| < 0.00002$?**
   - A. 1
   - B. 3
   - C. 5
   - D. 2
   - E. 4
   - F. 6

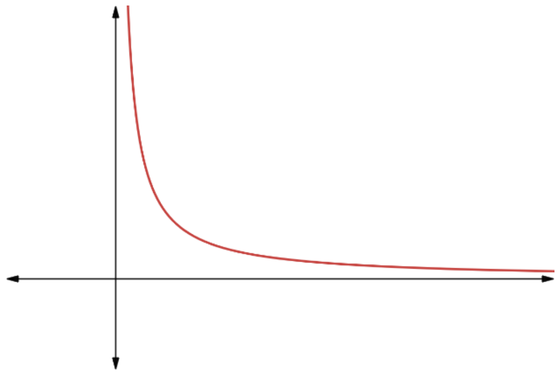
6. **The sum of the series $\sum_{n=1}^{\infty} (-1)^n \frac{\pi^{2n}}{3^{2n}(2n)!}$ is**
   - A. $\frac{\sqrt{3}}{2}$
   - B. 1
   - C. 0
   - D. $-\frac{1}{2}$
   - E. $-\frac{\sqrt{3}}{2}$
   - F. $\frac{1}{2}$

7. Consider the integral $\int_0^1 \frac{1}{x^p}\,dx$. For what values of $p > 0$ is the integral convergent?
   A. $0 < p < 1$ only
   B. $0 < p < 0.5$ only
   C. $0 < p \le 1$ only
   D. $1 < p < 2$ only
   E. $p > 1$ only
   F. $p > 2$ only

8. Applying the root test by evaluating $L = \lim_{n\to\infty} \sqrt[n]{|a_n|}$, the series $\sum_{n=1}^{\infty} \left(\frac{n^2+7}{3n^2+1}\right)^n$
   A. Diverges with $L = \frac{1}{3}$
   B. Converges with $L = \frac{1}{3}$
   C. Converges with $L = 3$
   D. Diverges with $L = \infty$
   E. Converges with $L = e^{-3}$
   F. Diverges with $L = 3$

9. Applying the ratio test by evaluating $L = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$, the series $\sum_{n=1}^{\infty} \frac{n\pi^n}{(-5)^{n-1}}$
   A. Converges with $L = \frac{\pi}{5}$
   B. Diverges with $L = \frac{5}{\pi}$
   C. Converges with $L = \frac{1}{5}$
   D. Diverges with $L = -\frac{\pi}{5}$
   E. Converges with $L = 0$
   F. Converges with $L = \frac{5}{\pi}$

10. The sum of the series $2 - \frac{4}{3} + \frac{8}{9} - \frac{16}{27} + \cdots$ is equal to
   A. 6
   B. 3
   C. $\frac{6}{5}$
   D. $\frac{3}{5}$
   E. 2
   F. None of the above



11. The area of the region enclosed between the curves $y = \frac{1}{3x}$, $y = 0, x = \frac{1}{2}$, and $x = 2$ is

A. $\frac{1}{6}$
B. 0
C. $\frac{1}{3} \ln 2$
D. $\frac{1}{2}$
E. $\frac{1}{3} \ln 4$
F. $-\frac{1}{3} \ln 2$

12. $\lim_{x \to \infty} \frac{x^{2023}}{e^x}$

A. $100!$
B. 0
C. $\frac{100!}{2^{100}}$
D. ∞
E. 1
F. Does not exist

13. If the radius of convergence of the power series $\sum_{n=1}^{\infty} a_n (x-3)^n$ is 2, then the series $\sum_{n=0}^{\infty} a_n$

A. converges conditionally
B. converges absolutely
C. cannot be determined
D. diverges
E. all of the above
F. none of the above

14. If the power series $\sum_{n=0}^{\infty} a_n x^n$ converges only in the interval $(-4, 4)$, then the series $\sum_{n=0}^{\infty} \frac{na_n}{2^n} x^n$ converges only in
    A. $(-4, 4)$
    B. $(-1, 1)$
    C. $(-8, 8)$
    D. $(-6, 6)$
    E. $(-\infty, \infty)$
    F. $(-2, 2)$

15. Find the Taylor series of $f(x) = \cos x$ centered at $a = \frac{\pi}{2}$
    A. $\sum_{n=0}^{\infty}(-1)^n \frac{1}{(2n+1)!}\left(x - \frac{\pi}{2}\right)^{2n+1}$
    B. $\sum_{n=0}^{\infty}(-1)^{n+1} \frac{1}{(2n+1)!}\left(x - \frac{\pi}{2}\right)^{2n+1}$
    C. $\sum_{n=0}^{\infty}(-1)^{n+1} \frac{1}{(2n)!}\left(x - \frac{\pi}{2}\right)^{2n}$
    D. $\sum_{n=1}^{\infty}(-1)^{n+1} \frac{1}{(2n+1)!}\left(x - \frac{\pi}{2}\right)^{2n+1}$
    E. $\sum_{n=0}^{\infty}(-1)^{n+1} \frac{1}{(2n+1)!} x^{2n+1}$
    F. $\sum_{n=0}^{\infty}(-1)^{n+1} \frac{1}{(2n+1)!}\left(x - \frac{\pi}{2}\right)^{2n}$

16. Compute the following limit by using L'Hospital Rule: $\lim_{t \to 0} \frac{e^{2t}-1}{\sin(t)}$.
    A. 2
    B. 1
    C. $\frac{1}{2}$
    D. 0
    E. -1
    F. $-\infty$

17. The substitution $u = tan(3x)$ transforms the integral $\int \frac{12\, sec^2(3x)}{tan^2(3x)+4\,tan(3x)+4}\, dx$ to:
   A. $\int \frac{12}{u^2+4u+4} du$
   B. $\int \frac{12(u^2+1)}{u^2+4u+4} du$
   C. $\int \frac{4}{u^2+4u+4} du$
   D. $\int \frac{36}{(u+2)^2} du$
   E. $\int \frac{36}{9u^2+12u+4} du$
   F. None of the above

18. Find the volume obtained from the following specifications.
   $y = x + 1, \quad y = 0, \quad x = 0, \quad x = 3;$ rotation about the x-axis.
   A. 6π
   B. 5π
   C. 3π
   D. 9π
   E. $\frac{26\pi}{3}$
   F. None of the above

19. Find the indefinite integral $\int sin^3(x)\, dx$. Hint: $\int sin^n(x)\, dx = -\frac{1}{n} sin^{n-1}(x)\, cos(x) + \frac{n-1}{n}\int sin^{n-2}(x)\, dx$.
   A. $-\frac{1}{3} sin^2(x)\, cos(x) - \frac{2}{3} cos(x) + C$
   B. $-\frac{1}{3} cos^2(x)\, sin(x) + \frac{2}{3} sin(x) + C$
   C. $\frac{1}{3} sin^2(x)\, cos(x) + \frac{2}{3} sin(x) + C$
   D. $-\frac{1}{3} cos^2(x)\, sin(x) - \frac{2}{3} sin(x) + C$
   E. $-\frac{2}{3} cos^2(x) - \frac{1}{3} sin^2(x) + C$
   F. None of the above

20. The sequence $\left\{\frac{n}{n+1}\right\}_{n=1}^{\infty}$ is:
   A. Divergent
   B. Decreasing
   C. Increasing
   D. Constant
   E. Unbounded
   F. None of the above is correct

21. The geometric series $2 + \frac{2}{3} + \frac{2}{9} + \cdots + \frac{2}{3^{n-1}} + \cdots$ converges to
   A. 0
   B. 1
   C. 2
   D. 3
   E. $\frac{7}{2}$
   F. None of the above

22. Use the root test to study the convergence of the series $\sum_{n=1}^{\infty}\left(\frac{n+3}{2n+2}\right)^{n}$.
   A. The series is absolutely convergent.
   B. The series is conditionally convergent.
   C. The series converges to $\frac{3}{2}$.
   D. The series is divergent.
   E. The series diverges to $-\infty$.
   F. None of the above statements is correct.

23. Determine the value of the improper integral $\int_0^{\infty} xe^{-x^2}\, dx$.
   A. ∞
   B. $-\infty$
   C. 1
   D. 0
   E. $\frac{1}{2}$
   F. None of the above

24. Consider the alternating series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}\sqrt{n}}{n+4}$. Use the appropriate test to study the convergence of the series.

- **A. The series is absolutely convergent.**
- **B. The series is conditionally convergent.**
- **C. The series converges to $\frac{1}{4}$.**
- **D. The series diverges to $-\infty$.**
- **E. The series diverges to $+\infty$.**
- **F. None of the above statements is correct.**

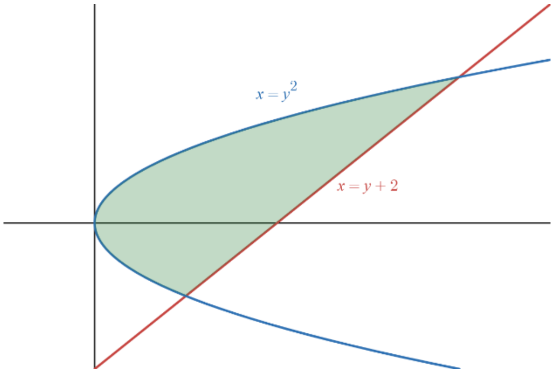
25. The interval of convergence of the power series $\sum_{k=1}^{\infty} \frac{(x-5)^k}{k^2}$ is given by:

- **A. $(-\infty, \infty)$**
- **B. $(-5, 5)$**
- **C. $(0, 5)$**
- **D. $[4, 6]$**
- **E. The empty interval**
- **F. None of the above**

26. The value of the telescopic series $\sum_{k=1}^{\infty} \frac{1}{k(k+1)}$ is:

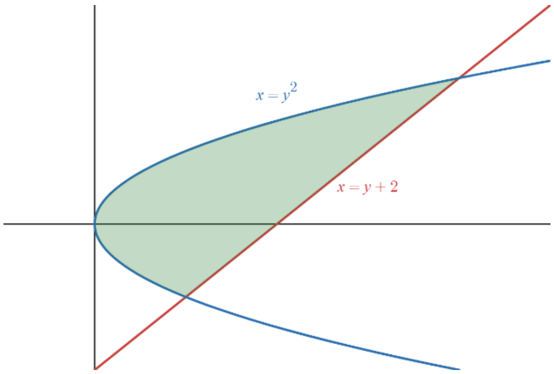
- **A. ∞**
- **B. e**
- **C. 0**
- **D. 1**
- **E. e^2**
- **F. None of the above**

27. Set up an integral that represents the volume of the solid generated by revolving the region bounded by $x = y^2$ and $x = y + 2$ about the line $y = 3$.



A. $\int_{-1}^{2}(3-y)(y+2-y^2)\,dy$
B. $\int_{-1}^{2} 2\pi(3-y)(y+2-y^2)\,dy$
C. $\int_{-2}^{1} 2\pi(3-y)(y^2-y-2)\,dy$
D. $\int_{1}^{4}(3-\sqrt{x})^2-(5-x)^2\,dx$
E. $\int_{-1}^{2} 2\pi(3+y)(y+2-y^2)\,dy$
F. $\pi\int_{1}^{4}(3-\sqrt{x})^2-(5-x)^2\,dx$

28. Set up an integral that represents the volume of the solid generated by revolving the region bounded by $x = y^2$ and $x = y + 2$ about the line $x = 5$.

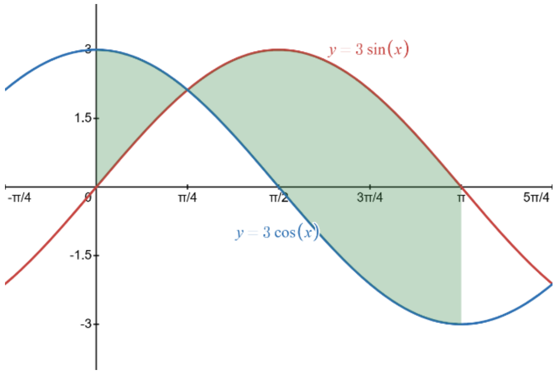


A. $\int_{-1}^{2}(5-y^2)^2-(3-y)^2\,dy$
B. $\pi\int_{-2}^{1}(5-y^2)^2-(3-y)^2\,dy$
C. $2\pi\int_{1}^{4}(5-x)(\sqrt{x}-x-2)\,dx$
D. $\pi\int_{-2}^{1}(5-y^2)^2-(5-y)^2\,dy$
E. $\int_{1}^{4}(5-x)(\sqrt{x}-x-2)\,dx$
F. $\pi\int_{-1}^{2}(5-y^2)^2-(3-y)^2\,dy$

29. Find the sum of the series $\sum_{n=2}^{\infty}\frac{1}{5^{n+1}}-\frac{1}{5^n}$.

A. 25
B. $\frac{1}{5}$
C. $-\frac{1}{25}$
D. $-\frac{1}{5}$
E. $\frac{1}{25}$
F. 0

30. The area of the shaded region between $y = 3\sin x$ and $y = 3\cos x$ is



A. $6\sqrt{2} + 12$
B. $12\sqrt{2}$
C. $3\sqrt{3} + 6$
D. $6\sqrt{2}$
E. $3\sqrt{2}$
F. 12

31. The radius and interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}5^n}$ are

A. $R = 5, [-5, 5]$
B. $R = 1, [-1, 1]$
C. $R = 5, [-5, 5)$
D. $R = 5, (-5, 5]$
E. $R = 5, (-5, 5)$
F. $R = 1, [-1, 1)$

32. Evaluate the integral $\int \frac{2}{x^2-1} dx$

A. $\ln|x-1| - \ln|x+1| + C$
B. $-\frac{2}{x-1} + C$
C. $\ln|x+1| - \ln|x-1| + C$
D. $2\ln|x-1| + 2\ln|x+1| + C$
E. $2\tan^{-1} x + C$
F. $2\tan^{-1}\left(\frac{x}{2}\right) + C$

33. The integral $\int \frac{x^2}{1+x^5}\,dx$ is represented as a power series as

A. $C+\sum_{n=0}^{\infty}(-1)^n \frac{x^{5n+1}}{5n+1}$

B. $C+\sum_{n=0}^{\infty}(-1)^{n+1} \frac{x^{5n+3}}{5n+3}$

C. $C+\sum_{n=0}^{\infty} \frac{x^{5n+3}}{5n+3}$

D. $C+\sum_{n=0}^{\infty}(-1)^n \frac{x^{5n+3}}{5n+3}$

E. $C+\sum_{n=0}^{\infty} \frac{x^{5n+2}}{5n+2}$

F. $C+\sum_{n=0}^{\infty}(-1)^n \frac{x^{5n}}{5n}$

34. Evaluate the integral $\int \frac{3^x}{\sqrt{1-9^x}}\,dx$.

A. $\frac{1}{\ln 3}\sin^{-1}(3^x)+C$

B. $-2\ln 3\sqrt{1-9^x}+C$

C. $(\ln 3)\sin^{-1}(3^x)+C$

D. $\sin^{-1}(3^x)+C$

E. $-\sqrt{1-9^x}+C$

F. $-2\sqrt{1-9^x}+C$

35. $\lim_{x\to 0^+} x^x =$

A. e

B. ∞

C. 1

D. 0

E. $\frac{1}{e}$

F. e^2

36. The sum of the series $\sum_{n=0}^{\infty}(-1)^n \frac{\pi^{2n+1}}{4^{2n+1}(2n+1)!}$ is
   A. -1
   B. 0
   C. 1
   D. $\frac{1}{2}$
   E. $\frac{\sqrt{3}}{2}$
   F. $\frac{1}{\sqrt{2}}$

37. The series $\sum_{n=1}^{\infty}\frac{1}{n^3}$ is convergent by the integral test. What is the least number of terms of the series we need to add in order to ensure that the sum is accurate within 0.0003?
   A. 40
   B. 21
   C. 58
   D. 41
   E. 42
   F. 10

38. Evaluate the integral $\int 3x^2 \ln x\, dx$.
   A. $x^3 \ln x + \frac{1}{3}x^3 + C$
   B. $x^3 \ln x - \frac{1}{3}x^3 + C$
   C. $x^3 \ln x - x^3 + C$
   D. $x^2 \ln x - \frac{1}{3}x^3 + C$
   E. $x^3 \ln x + x^3 + C$
   F. $x^2 \ln x - \frac{1}{2}x^3 + C$

39. Find the sum of the series $\sum_{n=1}^{\infty}\frac{2^{n+1}}{7^n}$.
   A. 5
   B. $\frac{2}{7}$
   C. 15
   D. $\frac{2}{5}$
   E. $\frac{4}{5}$
   F. $\frac{4}{7}$

Written Questions.

1. **Evaluate the integral.**

$$\int \sqrt{\sin x}\, \cos^3 x\, dx$$

2. **Find the interval of convergence of the power series.**

$$\sum_{n=1}^{\infty} \frac{(-1)^n 3^n}{\sqrt{n}} x^n$$

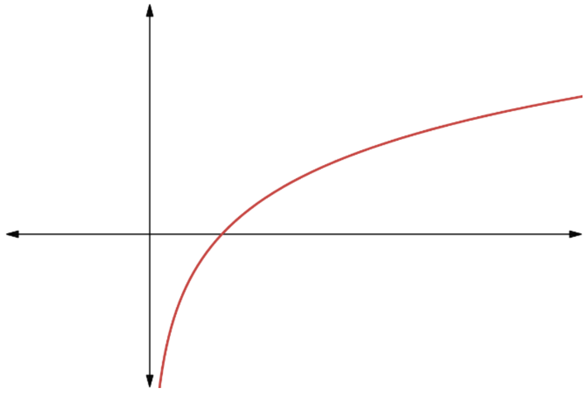
3. **Determine whether the series is convergent or divergent.**

$$\sum_{n=1}^{\infty} \frac{\sqrt{3n^3+5}}{7n^2-1}$$

4. **Determine whether the improper integral is convergent or divergent.**

$$\int_1^{\infty} \frac{\arctan x}{2+e^x} dx$$

5. **Write an integral that gives the volume of the solid generated by revolving the region bounded by $y = \ln x$ and the x-axis around the y-axis for $1 \le x \le e^3$. Evaluate the integral.**



6. Find the sum of the following series:

$$\sum_{n=0}^{\infty} \frac{2^n - (-2)^n}{e^n}$$

7. Let $f(x) = \frac{1}{1+2x^3}$.

(a) Express $f(x)$ as a power series.

(b) For which x does the above series converges?

(c) Use the power series to compute $f^{(6)}(0)$.

8. Find the area of the region bounded by the parabola $y = x^2$, and the line $y = 2$.

9. Evaluate by partial fractions the indefinite integral $\int \frac{x+4}{x^2-5x+6}dx$

10. Use the Ratio Test to decide if the series is convergent or divergent.

$$\sum_{n=0}^{\infty} \frac{5^{-n}}{n+1}$$

What are the other tests you can use to decide about the convergence or the divergence of the above series?

11. Write the power series representation for the function $f(x) = \frac{1}{1+x}$. Differentiate the power series for $\frac{1}{1+x}$ term by term to find the sum $\sum_{n=1}^{\infty}(-1)^n n x^{n-1}$ for $|x| < 1$.

12. Determine if the series $\sum_{n=2}^{\infty}\frac{(-1)^n}{n\sqrt[3]{\ln n}}$ is absolutely convergent, conditionally convergent or divergent.

13. Determine if the series converges or diverges.

(a) $\sum_{n=1}^{\infty}\frac{3n^3+2n+6}{7n^4+5n+1}$

(b) $\sum_{n=1}^{\infty} \frac{5^n n^2}{(2n+1)!}$

(c) $\sum_{n=1}^{\infty} \frac{\cos n}{n^5+5}$

14. Determine if the improper integral $\int_1^{\infty} \frac{1+\cos^2 x}{x+1}\,dx$ is convergent or divergent.

15. Evaluate the integral $\int \sqrt{16 - x^2}\, dx$.