Multiple Choice Questions.

1. $\int x^2 \sin(3x)\,dx =$

   A. $-\frac{x^2}{3}\cos 3x + \frac{2}{9}x\sin 3x + \frac{2}{27}\cos 3x + C$

   B. $\frac{x^2}{3}\cos 3x + \frac{2}{9}x\sin 3x + \frac{2}{27}\cos 3x + C$

   C. $-\frac{x^2}{3}\cos 3x + \frac{2}{9}x\sin 3x - \frac{2}{27}\cos 3x + C$

   D. $\sin 3x\left(-\frac{x^2}{3} + \frac{2}{9}x + \frac{2}{27}\right) + C$

   E. $\cos 3x\left(-\frac{x^2}{3} + \frac{2}{9}x + \frac{2}{27}\right) + C$

   F. None of these

2. Using the integration by parts method, evaluate the indefinite integral $\int x\cos(7x)\,dx$

   A. $\frac{1}{7}x\cos(7x) + \frac{1}{49}\sin(7x) + C$

   B. $\frac{1}{7}x\sin(7x) + \frac{1}{49}\cos(7x) + C$

   C. $x\sin(7x) + \cos(7x) + C$

   D. $x\cos(7x) + \sin(7x) + C$

   E. $\frac{1}{49}x\sin(7x) + \frac{1}{7}\cos(7x) + C$

   F. None of the above.

3. $\int x\cos^5(x^2)\,dx =$

   A. $\frac{1}{2}\sin(x^2) - \frac{1}{3}\sin^3(x^2) + \frac{1}{10}\sin^5(x^2) + C$

   B. $\frac{1}{2}\sin(x^2) + \frac{1}{3}\sin^5(x^2) + \frac{1}{10}\sin^3(x^2) + C$

   C. $\frac{1}{2}\cos(x^2) - \frac{1}{3}\cos^2(x^2) + \frac{1}{10}\sin^4(x^2) + C$

   D. $\frac{1}{2}\cos(x^2) - \frac{1}{3}\cos^4(x^2) + \frac{1}{10}\sin^2(x^2) + C$

   E. $\frac{1}{2}\cos(x^2) - \frac{1}{3}\cos^3(x^2) + \frac{1}{10}\cos^5(x^2) + C$

   F. $\frac{1}{2}\cos(x^2) - \frac{1}{3}\cos^3(x^2) + \frac{1}{5}\cos^5(x^2) + C$

4. Suppose that $f(1) = 2, f(4) = 7, f'(1) = 5$ and $f'(4) = 3$. We assume that f'' is a continuous function. Find the definite integral $\int_1^4 x f''(x)\, dx$.

A. 4
B. 3
C. 2
D. 1
E. 0.5
F. 0

5. Using integration by parts once, the integral $\int (\ln x)^{2023}\, dx$ becomes

A. $2023\frac{(\ln x)^{2022}}{x} + C$
B. $x(\ln x)^{2023} - \int (\ln x)^{2022}\, dx$
C. $2023 \int (\ln x)^{2022}\, dx$
D. $x(\ln x)^{2023} - 2023\int (\ln x)^{2022}\, dx$
E. $\frac{1}{2024}(\ln x)^{2024} + C$
F. None of these

6. $\int x^2 e^{-3x}\, dx =$

A. $-\frac{1}{27}e^{-3x}\left(9x^2 + 6x + 2\right) + C$
B. $e^{-3x}\left(-\frac{x^2}{3} + \frac{2x}{9}\right) + C$
C. $-\frac{1}{27}e^{-3x}\left(9x^2 - 6x + 2\right) + C$
D. $-\frac{1}{27}e^{-3x}\left(9x^2 - 6x - 2\right) + C$
E. $e^{-3x}\left(9x^2 + 6x + 2\right) + C$
F. None of these

7. $\int \tan^2 x \cos^3 x\, dx =$

A. $\frac{(\sin x)^3}{3} + C$
B. $-\frac{(\sin x)^3}{3} + C$
C. $-\frac{(\cos x)^3}{3} + C$
D. $\frac{(\cos x)^3}{3} + C$

8. $\int x^2 \ln x \, dx$
   A. $\frac{x^3 \ln x}{3} - \frac{x^3}{3} + C$
   B. $\frac{x^3 \ln x}{3} - \frac{x^3}{9} + C$
   C. $\frac{x^4 \ln x}{4} - \frac{x^3}{4} + C$
   D. $\frac{x^4 \ln x}{4} - \frac{x^3}{16} + C$

9. Evaluate the integral $\int 9x \sin(3x) \, dx$.
   A. $-3x\cos(3x) + \sin(3x) + C$
   B. $-3\cos(3x) - \sin(3x) + C$
   C. $-3x^2\cos(3x) + C$
   D. $3x\cos(3x) + C$
   E. $-3x\cos(3x) + 9\sin(3x) + C$
   F. $3x\cos(3x) + \sin(3x) + C$

10. Evaluate the integral $\int \sin^{-1} x \, dx$.
   A. $x\sin^{-1} x + 2\sqrt{1-x^2} + C$
   B. $x\sin^{-1} x - \sqrt{1-x^2} + C$
   C. $x\sin^{-1} x + \sqrt{1-x^2} + C$
   D. $\sqrt{1-x^2} + C$
   E. $x\sin^{-1} x - \frac{1}{2}\sqrt{1-x^2} + C$

11. Evaluate the integral $\int \csc^4 x \, dx$.
   A. $-\frac{1}{3}\cot^3 x - \cot x + C$
   B. $\frac{1}{3}\cot^3 x + \cot x + C$
   C. $-\cot^3 x - \cot x + C$
   D. $\frac{1}{5}\csc^5 x + C$
   E. $\frac{1}{3}\cot^3 x + C$
   F. $-\cot^3 x + x + C$

12. $\int 5x^9 e^{x^5}\, dx =$
   A. $\frac{5x^7 e^{x^5}}{7}$
   B. $e^{x^5} + C$
   C. $x^5 e^{x^5} - e^{x^5} + C$
   D. $x^6 e^{x^5} + C$
   E. $x^{5x^6} e^{x^5} + C$
   F. $xe^x - e^x + C$

13. $\int \sin^2 x \cos^2 x\, dx =$
   A. $\frac{1}{4}\sin^2(2x) + C$
   B. $\frac{1}{3}\sin^3 x \cos x + C$
   C. $\frac{1}{3}\sin^3 x - \frac{1}{5}\sin^5 x + C$
   D. $\frac{1}{8}x - \frac{1}{32}\sin(4x) + C$
   E. None of the above

14. If the indefinite integral $\int \sin^2 x \cos^2 x\, dx = Ax + B\sin(4x) + C$, then
   A. $A = \frac{1}{2}, B = \frac{1}{8}$
   B. $A = \frac{1}{2}, B = -\frac{1}{8}$
   C. $A = \frac{1}{8}, B = \frac{1}{32}$
   D. $A = -\frac{1}{8}, B = \frac{1}{32}$
   E. $A = \frac{1}{8}, B = -\frac{1}{32}$
   F. None of these

15. $\int \tan^3 x \sec x\, dx =$
   A. $\frac{1}{3}\sec^3 x - \sec x + C$
   B. $\frac{1}{4}\tan^4 x + C$
   C. $\frac{1}{2}\tan x \sec^2 x + C$
   D. $\ln|\tan x + \sec x| + C$
   E. None of the above

16. Find the indefinite integral $\int tan(11x)\, sec^{20}(11x)\, dx$.
    A. $\frac{sec^{21}(11x)}{231} + C$
    B. $\frac{sec^{20}(11x)}{220} + C$
    C. $\frac{sec^{20}(11x)\, tan^{21}(11x)}{220} + C$
    D. $\frac{sec^{11}(11x)}{220} + C$
    E. $sec^{21}(11x) + C$
    F. None of these

17. Which of the following is equivalent to the integral $\int_0^{\frac{\pi}{2}} cos^9 x\, sin^5 x\, dx$?
    A. $\int_0^{\frac{\pi}{2}} [cos^9 x \sin x - 2\, cos^{11} x \sin x - cos^{13} x \sin x]\, dx$
    B. $\int_0^{\frac{\pi}{2}} [cos^9 x \sin x + 2\, cos^{11} x \sin x + cos^{13} x \sin x]\, dx$
    C. $\int_0^{\frac{\pi}{2}} [cos^{14} x \sin x + 2\, cos^{11} x \sin x - cos^9 x \sin x]\, dx$
    D. $\int_0^{\frac{\pi}{2}} [cos^9 x \sin x - 2\, cos^{11} x \sin x + cos^{13} x \sin x]\, dx$
    E. $\int_0^{\frac{\pi}{2}} [cos^9 x \sin x - 2\, cos^{10} x \sin x + cos^{11} x \sin x]\, dx$
    F. None of these

18. The value of the integral $\int_0^2 \frac{x^2 - x}{x+1}\, dx$ is
    A. 4
    B. $ln(3) + 2$
    C. $-2[1 - ln(3)]$
    D. $ln(3) - 2$
    E. $-2[ln(3) - 2]$
    F. None of these

19. Find the indefinite integral $\int \frac{1}{\sqrt{x^2-4}}\, dx$.
    A. $\sqrt{x^2 - 4} + C$
    B. $ln\left(\sqrt{x^2 - 4}\right) + C$
    C. $ln\left|x + \sqrt{x^2 - 4}\right| + C$
    D. $ln|x| + C$
    E. $ln|x| + \sqrt{x^2 - 4} + C$
    F. None of these

20. **Which of these substitutions will facilitate evaluating the integral $\int_0^4 \frac{1}{\sqrt{4x-x^2}}dx$?**
   A. $x = 2 + 2\sin t$
   B. $x = 2\cos t$
   C. $x = 2\sin t$
   D. $x = 2\sec t$
   E. $x = \tan t$
   F. $x = \sin t$

21. **Which of these substitutions will help evaluate the integral $\int \frac{\sqrt{16-9\theta^2}}{\theta^2}d\theta$?**
   A. $x = \frac{3}{4}\sin\theta$
   B. $x = \frac{3}{4}\tan\theta$
   C. $x = \frac{4}{3}\sec\theta$
   D. $x = \frac{4}{3}\sin\theta$
   E. $x = 3\sin\theta$
   F. None of these

22. **Which of these substitutions will facilitate evaluating the integral $\int \frac{1}{\sqrt{8t-t^2}}dt$?**
   A. $t = \sin^{-1}(8-\theta)$
   B. $t = 4(1+\sin\theta)$
   C. $t = 4\sin\theta$
   D. $t = 4\sec\theta$
   E. $t = \tan\theta$
   F. $t = \cos\theta$

23. **The partial fraction decomposition of the function $\frac{x^2-14x-17}{(x-1)^2(x^2+1)}$ is of the form**
   A. $\frac{A}{(x-1)^2} + \frac{B}{x^2+1}$
   B. $\frac{A}{(x-1)^2} + \frac{Bx+C}{x^2+1}$
   C. $\frac{Ax+B}{x-1} + \frac{Cx+D}{x^2+1}$
   D. $\frac{Ax+B}{x-1} + \frac{Cx+D}{(x-1)^2} + \frac{Ex+F}{x^2+1}$
   E. $\frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{Cx+D}{x^2+1}$

24. The partial fraction decomposition of the function $\frac{x^3+x^2+1}{x^2(x^2+x+1)(x^2+1)^2}$ is

A. $\frac{A}{x^2}+\frac{B}{x^x+x+1}+\frac{C}{(x^2+1)^2}$
B. $\frac{A}{x}+\frac{B}{x^2}+\frac{Cx+D}{x^2+x+1}+\frac{Ex+F}{x^2+1}+\frac{Gx+H}{(x^2+1)^2}$
C. $\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x^2+x+1}+\frac{D}{x^2+1}+\frac{E}{(x^2+1)^2}$
D. $\frac{A}{x}+\frac{B}{x^2}+\frac{Cx+D}{x^2+x+1}+\frac{Ex+F}{(x^2+1)^2}$
E. None of the above

25. Which of these is the partial fraction decomposition for the integrand in $\int \frac{x^3-2x^2+2x-5}{x^4+4x^2+3}dx$?

A. $\frac{A}{x-1}+\frac{B}{x+1}+\frac{Cx+D}{x^2+3}$
B. $\frac{A}{x-3}+\frac{B}{x+3}+\frac{Cx+D}{x^2+1}$
C. $\frac{Ax+B}{x^2+1}+\frac{Cx+D}{x^2+3}$
D. $\frac{Ax+B}{x^2-1}+\frac{Cx+D}{x^2+3}$
E. $\frac{Ax+B}{x^2+1}+\frac{Cx+D}{x^2-3}$
F. None of these

26. Which of these is a partial fraction decomposition for the expression $\frac{x^3+x^2+1}{(x^2+x+1)(x^2+1)^3(x-1)x}$?

A. $\frac{A}{x^2+x+1}+\frac{B}{x^2+1}+\frac{C}{(x^2+1)^2}+\frac{D}{(x^2+1)^3}+\frac{E}{x-1}+\frac{F}{x}$
B. $\frac{Ax+B}{x^2+x+1}+\frac{Cx+D}{(x^2+1)^3}+\frac{E}{x-1}+\frac{F}{x}$
C. $\frac{A}{x^2+x+1}+\frac{Bx+C}{x^2+1}+\frac{Dx+E}{(x^2+1)^2}+\frac{Fx+G}{(x^2+1)^3}+\frac{H}{x-1}+\frac{K}{x}$
D. $\frac{Ax+B}{x^2+x+1}+\frac{Cx+D}{x^2+1}+\frac{Ex+F}{(x^2+1)^2}+\frac{G}{x-1}+\frac{H}{x}$
E. $\frac{Ax+B}{x^2+x+1}+\frac{C}{x^2+1}+\frac{D}{(x^2+1)^2}+\frac{E}{(x^2+1)^3}+\frac{F}{x-1}+\frac{G}{x}$
F. $\frac{Ax+B}{x^2+x+1}+\frac{Cx+D}{x^2+1}+\frac{Ex+F}{(x^2+1)^2}+\frac{Gx+H}{(x^2+1)^3}+\frac{I}{x-1}+\frac{K}{x}$

27. Which of these is a partial fraction decomposition for the integrand in the integral $\int \frac{4-2x}{(x^2+1)^2(x-1)^2}dx$?

A. $\frac{Ax+B}{x^2+1}+\frac{C}{x-1}$
B. $\frac{Ax+B}{x^2+1}+\frac{Cx+D}{x-1}$
C. $\frac{Ax+B}{x^2+1}+\frac{C}{x-1}+\frac{D}{(x-1)^2}$
D. $\frac{Ax+B}{x^2+1}+\frac{Cx+D}{(x^2+1)^2}+\frac{E}{x-1}+\frac{F}{(x-1)^2}$
E. $\frac{A}{x^2+1}+\frac{B}{(x^2+1)^2}+\frac{C}{x-1}+\frac{D}{(x-1)^2}$
F. $\frac{Ax+B}{(x^2+1)^2}+\frac{C}{(x-1)^2}$

28. If the indefinite integral $\int \frac{x-4}{x^2+2x}dx = A\,ln|x| - B\,ln|x+2| + C$, then

A. $A = -2\,, B = -3$
B. $A = 2\,, B = -3$
C. $A = 2\,, B = 3$
D. $A = -2\,, B = 3$
E. $A = -B = 2$
F. None of these

29. $\int \frac{x^3+4}{x^2+4}dx =$

A. $\frac{x^2}{2} + C$
B. $\frac{x^2}{2} + 2\,ln|x^2+4| + C$
C. $\frac{x^2}{2} - 2\,ln|x^2+4| + 2\,tan^{-1}\left(\frac{x}{2}\right) + C$
D. $\frac{x^2}{2} - 4\,ln|x^2+4| + 2\,tan^{-1}\left(\frac{x}{2}\right) + C$
E. $\frac{x^2}{2} - 2\,ln|x^2+4| + 4\,tan^{-1}\left(\frac{x}{2}\right) + C$
F. None of the above

30. $\int \frac{3x}{x^2+2x-8}dx =$

A. $ln\left|\frac{(x-2)^2}{x+4}\right| + C$
B. $ln\left|\frac{x+2}{(x+4)^2}\right| + C$
C. $ln\left|\frac{x-2}{(x+4)^2}\right| + C$
D. $ln\left|\frac{x+6}{x+4}\right| + C$
E. $ln\left|\frac{x-2}{x+6}\right| + C$
F. None of the above

31. The approximate value of the integral $\int_7^9 \frac{\sin x}{x}dx$ using Trapezoidal Rule with $n = 4$ is

A. 0.212546
B. 0.210502
C. 0.206250
D. 0.210443
E. 0.412499

32. Using Simpson's Rule with $n = 4$ to approximate the integral $\int_0^1 \sqrt{1+x^3}\,dx$ to six decimal places gives
   A. 1.108667
   B. 1.116993
   C. 1.111363
   D. 1.111448
   E. None of the above

33. Given the functional values in the table, use Simpson's Rule to approximate $\int_2^5 f(x)\,dx$ to 2 decimal places.

x	2.0	2.5	3.0	3.5	4.0	4.5	5.0
$f(x)$	3.2	2.7	4.1	3.8	3.5	4.6	5.2

   A. 11.45
   B. 10.33
   C. 13.55
   D. 11.33
   E. 10.38
   F. None of these

34. Use the Midpoint Rule with $n = 5$ to approximate the integral $\int_1^2 \frac{1}{x}\,dx$ to 2 decimal places.
   A. 0.68
   B. 0.69
   C. 0.70
   D. 0.71
   E. 0.72
   F. None of these

35. For what values of p does the integral $\int_0^1 \frac{8}{x^p}\,dx$ converges?
   A. $p < 1$
   B. $p \leq 1$
   C. $p > 1$
   D. $p \geq 1$
   E. $-1 < p < 1$

36. For which of these values of k does the integral $\int_1^{\infty} \frac{1}{y^k} dy$ diverges?

A. $k > 1$
B. $k \leq 1$
C. $k \geq 1$
D. $0 < k < 1$
E. $0 \leq k < 1$
F. $-1 \leq k \leq 1$

37. The integral $\int_{-\infty}^{0} xe^{-x^2} dx$ is

A. convergent and equal to $-\frac{1}{2}$
B. convergent and equal to 0
C. convergent and equal to 1
D. divergent
E. None of the above

38. The integral $\int_0^3 \frac{1}{x-1} dx$ is

A. convergent and equal to 0
B. convergent and equal to 1
C. convergent and equal to $ln\, 2$
D. convergent and equal to $2\, ln\, 2$
E. None of the above

39. The integral $\int_2^{\infty} \frac{1}{x-\ln x} dx$

A. diverges
B. converges to 0
C. converges to $-\, ln\, 2$
D. converges to $-\, ln \frac{1}{3}$
E. converges to $ln \frac{1}{3}$
F. converges to $ln\, 2$

40. The improper integral $\int_1^{\infty} \frac{1}{\sqrt{x^2-0.1}} dx$

A. **Converges to** $sin^{-1}\left(\frac{1}{10}\right)$
B. **Converges to** $sin^{-1}(5)$
C. **Converges to** $sec^{-1}\left(\frac{1}{5}\right)$
D. **Converges to** $sec^{-1}(2)$
E. **Converges to** $tan^{-1}\left(\frac{1}{2}\right)$
F. **Diverges**

41. Evaluate the integral $\int x\, csc^2\, x\, dx$

A. $x \cot x - ln|sin\, x| + C$
B. $-x \cot x + csc^2\, x + C$
C. $-x \cot x + \frac{1}{2} cot^2\, x + C$
D. $-x \cot x - csc^2\, x + C$
E. $-x \cot x + ln|sin\, x| + C$
F. $x \cot x + cot^2\, x + C$

Written Questions.

1. **Evaluate the integral.**

$$\int \frac{x^2}{\sqrt{9 - x^2}} dx$$

2. Evaluate the integral.

$$\int \frac{\sqrt{9-x^2}}{x^2}\,dx$$

3. Evaluate the integral.

$$\int \frac{10}{(x-1)(x^2+9)}\,dx$$

4. Evaluate the integral $\int x\, tan^{-1}(x)\, dx$.

5. Evaluate the integral $\int x^2 \, sin\, x\, dx$.

6. Evaluate the integral $\int sin(ln\, x)\, dx$.

7. **Evaluate the indefinite integral $\int \ln(\sqrt{x})\, dx$.**

8. **Evaluate the integral $\int t\cos^5(t^2)\, dt$.**

9. **Evaluate the integral $\int e^x \cos x\, dx$.**

10. Evaluate the integral $\int \sin(\sqrt{x})\, dx$.

11. Use the below tabulated values of the function $f(x)$ to estimate the integral $\int_0^1 f(x)\, dx$, using Simpson's rule. Round your answer to 3 decimal places.

x	$f(x)$	
0.000	0.000	
0.125	0.124	
0.250	0.242	
0.375	0.347	
0.500	0.433	
0.625	0.487	
0.750	0.496	
0.875	0.423	
1.000	0.000	

12. Evaluate the following integral.

$$\int \frac{2x^2 - 2x + 4}{x^3 + 4x}\, dx$$

13. By considering the substitution $u = \tan\theta$, evaluate the integral.

$$\int \tan^4\theta \, d\theta$$

14. Evaluate the integral.

$$\int \frac{1}{6+e^x}\,dx$$

15. Use Simpson's Rule with $n = 6$ to estimate (up to 3 decimal places) the value of the integral.

$$\int_0^1 \sqrt{x^3+x}\,dx$$

16. Suppose $f(0) = f'(1) = 1, f(1) = f'(0) = 2$. Assume that f'' is continuous on $[0, 1]$. Evaluate $\int_0^1 x f''(x)\, dx$.

17. Show that $\int_0^{\ln 4} \frac{e^x}{\sqrt{e^{2x}+9}}\, dx = \ln(9) - \ln\left(1 + \sqrt{10}\right)$.

18. Evaluate the following integral $\int_1^{+\infty} \frac{2x-1}{x^3+x} dx$ if it is convergent.

19. Evaluate the trigonometric integral

$$\int tan^4 \theta \, d\theta$$

20. Determine whether each of the following improper integral converges or diverges. If it converges, find its value.

(a) $\int_0^4 \frac{1}{x^2+x-2} dx$

(b) $\int_0^4 \frac{1}{x^2-x-2}\,dx$

(c) $\int_{-1}^0 \frac{e^{\frac{1}{x}}}{x^3}\,dx$

(d) $\int_4^\infty \frac{1}{(x-2)(x-3)}\,dx$

(e) $\int_4^{\infty} \frac{1}{(x+2)(x-3)} dx$

(f) $\int_2^{\infty} \frac{1}{\sqrt{x}-1} dx$

(g) $\int_0^{\infty} \frac{1}{x^4+1} dx$

21. Evaluate the integral

$$\int e^{-x}\, ln\left(1 + e^{2x}\right) dx$$