



MATHS102 Calculus II

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التسجيل لمراجعة Test1

يشمل فيديوهات الشرح التفصيلي والنوتات لجميع الدروس
بالإضافة إلى حل أسئلة امتحانات سابقة



سجل الآن
عبر موقعنا



Multiple Choice Questions.

1. The value of $\lim_{x \rightarrow 0^+} \sqrt{x} (\ln x)^2$ is

- A. ∞
- B. $-\infty$
- C. 0
- D. -1
- E. 1
- F. None of these

2. The value of $\lim_{x \rightarrow 0} \frac{x - \sin x}{x - \tan x}$ is

- A. 0
- B. ∞
- C. 1
- D. -2
- E. $-\frac{1}{2}$
- F. None of these

3. Up to some constant C the indefinite integral $\int \frac{\ln x}{x(\ln x - 1)} dx$ is

- A. $\ln \left| \frac{x-1}{x} \right|$
- B. $\ln \left| \frac{x}{x-1} \right|$
- C. $\ln|\ln x - 1|$
- D. $\ln|x(\ln x - 1)|$
- E. $\ln|\ln x - x|$
- F. None of these

4. Suppose $f(1) = f'(2) = 1, f(2) = f'(1) = 2$. Assume that f'' is continuous on $[1, 2]$. The definite integral $\int_1^2 xf''(x) dx$ is

- A. 0
- B. -1
- C. 1
- D. -2
- E. 2
- F. None of these

5. Which of these is the volume of the solid formed by revolving the region bounded by the graphs $y = x$ and $y^2 = x$ about the y-axis?
- A. $\frac{2\pi}{15}$
 B. $\frac{\pi}{6}$
 C. $\frac{\pi}{2}$
 D. $\frac{\pi}{15}$
 E. $\frac{\pi}{3}$
 F. None of these
6. $\lim_{x \rightarrow +\infty} e^{2x} - x \ln x =$
- A. 0
 B. $-\infty$
 C. 1
 D. e
 E. $+\infty$
 F. None of these
7. The area of the region bounded by $y = -x^2$ and $y = x^2 - 1$ is
- A. $\frac{2\sqrt{2}}{3}$
 B. $\frac{\sqrt{2}}{3}$
 C. $\frac{2\sqrt{2}}{4}$
 D. $\frac{\sqrt{3}}{4}$
 E. $2\sqrt{3}$
 F. None of these
8. $\int_{-1}^0 xe^{-x^2} dx =$
- A. $\frac{1+e}{2e}$
 B. $\frac{e-1}{2e}$
 C. $\frac{1}{2e}$
 D. $\frac{1-e}{2e}$
 E. $\frac{2e}{e+1}$
 F. None of these

9. $\int x^2 \sin(3x) dx =$

- A. $-\frac{x^2}{3} \cos 3x + \frac{2}{9}x \sin 3x + \frac{2}{27} \cos 3x + C$
- B. $\frac{x^2}{3} \cos 3x + \frac{2}{9}x \sin 3x + \frac{2}{27} \cos 3x + C$
- C. $-\frac{x^2}{3} \cos 3x + \frac{2}{9}x \sin 3x - \frac{2}{27} \cos 3x + C$
- D. $\sin 3x \left(-\frac{x^2}{3} + \frac{2}{9}x + \frac{2}{27}\right) + C$
- E. $\cos 3x \left(-\frac{x^2}{3} + \frac{2}{9}x + \frac{2}{27}\right) + C$
- F. None of these

10. The value of $\lim_{x \rightarrow (\frac{\pi}{2})^-} \frac{\sec x}{\tan x}$ is

- A. $-\infty$
- B. $+\infty$
- C. 1
- D. 0
- E. -1
- F. None of these

11. The value of $\lim_{x \rightarrow 1} \frac{\ln(x)}{\sin(\pi x)}$ is

- A. 0
- B. $-\infty$
- C. $-\frac{1}{\pi}$
- D. π
- E. $-\pi$
- F. None of the above.

12. Compute the following limit by using L'Hospital Rule: $\lim_{t \rightarrow 1} \left[\frac{t}{t-1} - \frac{1}{\ln(t)} \right]$.

- A. 1
- B. $\frac{1}{2}$
- C. ∞
- D. $-\infty$
- E. 0
- F. -1

13. Find the indefinite integral $\int \sin(\theta) \sqrt{1 + \cos(\theta)} d\theta$.

- A. $-\frac{2}{3}(1 + \cos(\theta))^{3/2} + C$
- B. $\frac{3}{2}(1 + \sin(\theta))^{2/3} + C$
- C. $-2(1 + \cos(\theta))^{1/2} + C$
- D. $3(1 + \sin(\theta))^{1/3} + C$
- E. $-\frac{2}{3}(1 + \sin(\theta))^{3/2} + C$
- F. None of the above.

14. Find the definite integral $\int_0^1 x\sqrt{1 - x^2} dx$.

- A. $\frac{3}{2}$
- B. $\sqrt{2}$
- C. 1
- D. $\frac{\sqrt{2}}{2}$
- E. $\frac{1}{4}$
- F. $\frac{1}{3}$

15. Compute the indefinite integral $\int \frac{e^{\sin^{-1}(x)}}{\sqrt{1-x^2}} dx$.

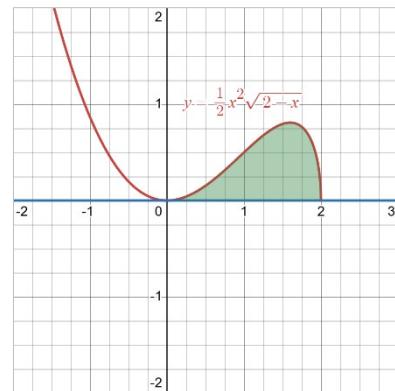
- A. $\sin^{-1}(x) + C$
- B. $e^{\sin^{-1}(x)} + C$
- C. $\tan^{-1}(x) + C$
- D. $e^{\tan^{-1}(x)} + C$
- E. $\cos^{-1}(x) + C$
- F. $e^{\cos^{-1}(x)} + C$

16. Use the appropriate method to find the volume of the solid formed by revolving the region bounded by the graphs of $y = \sqrt{x}$ and $y = x^2$ about the y-axis.

- A. 2π
- B. π
- C. $\frac{\pi}{2}$
- D. $\frac{\pi}{4}$
- E. $\frac{3\pi}{10}$
- F. $\frac{\pi}{5}$

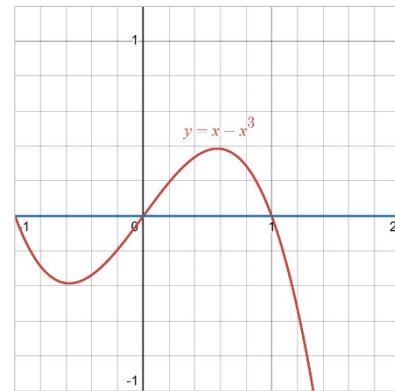
17. A tank on the wing of a jet aircraft is formed by revolving the region bounded by the graph of $y = \frac{1}{2}x^2\sqrt{2-x}$ and the x-axis about the x-axis, where x and y are measured in meters. Find the tank's volume.

- A. 6π
- B. 5π
- C. 3π
- D. $\frac{8\pi}{15}$
- E. $\frac{16\pi}{3}$
- F. None of the above.



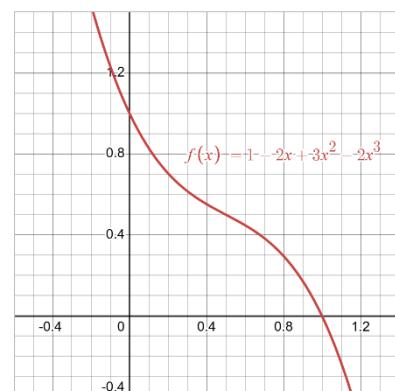
18. Use the shell method to find the volume of revolution formed by revolving the region bounded by $y = x - x^3$ and the x-axis about the y-axis. The most accurate answer is

- A. $\frac{5\pi}{9}$
- B. π
- C. $\frac{5\pi}{3}$
- D. $\frac{8\pi}{9}$
- E. $\frac{4\pi}{15}$
- F. $\frac{3\pi}{2}$



19. Use the appropriate method to find the volume of the solid obtained by rotating the area under the graph of $f(x) = 1 - 2x + 3x^2 - 2x^3$ and the x-axis over $[0, 1]$ about the y-axis. The most accurate answer is

- A. $\frac{3\pi}{4}$
- B. π
- C. $\frac{4\pi}{3}$
- D. 2π
- E. $\frac{11\pi}{30}$
- F. $\frac{3\pi}{2}$



20. Using the integration by parts method, evaluate the indefinite integral $\int x \cos(7x) dx$

- A. $\frac{1}{7}x \cos(7x) + \frac{1}{49}\sin(7x) + C$
- B. $\frac{1}{7}x \sin(7x) + \frac{1}{49}\cos(7x) + C$
- C. $x \sin(7x) + \cos(7x) + C$
- D. $x \cos(7x) + \sin(7x) + C$
- E. $\frac{1}{49}x \sin(7x) + \frac{1}{7}\cos(7x) + C$
- F. None of the above.

21. Consider the two functions y_1 and y_2 defined by $y_1 = x^2$ and $y_2 = -x^2 + 1$. Determine the area enclosed by the graphs of y_1 and y_2 .

- A. $\frac{2\sqrt{2}}{3}$
- B. $\frac{\sqrt{2}}{3}$
- C. $\frac{2\sqrt{3}}{4}$
- D. $\frac{\sqrt{3}}{4}$
- E. $2\sqrt{3}$
- F. None of the above

22. Suppose that $f(1) = 2, f(4) = 7, f'(1) = 5$ and $f'(4) = 3$. We assume that f'' is a continuous function. Find the definite integral $\int_1^4 xf''(x) dx$.

- A. 4
- B. 3
- C. 2
- D. 1
- E. 0.5
- F. 0

23. The value of $\lim_{x \rightarrow 0^+} \sqrt{x} \ln x$ is

- A. ∞
- B. $-\infty$
- C. 0
- D. -1
- E. 1
- F. None of these

24. The value of $\lim_{x \rightarrow 0} \frac{x^3}{\tan x - x}$ is

- A. 0
- B. ∞
- C. 1
- D. 3
- E. $\frac{1}{3}$
- F. None of these

25. Up to some constant C the indefinite integral $\int \frac{dx}{x(\ln x - 1)}$ is

- A. $\ln \left| \frac{x-1}{x} \right|$
- B. $\ln \left| \frac{x}{x-1} \right|$
- C. $\ln |\ln x - 1|$
- D. $\ln |\ln x - x|$
- E. $\ln |\ln x| - x$
- F. None of these

26. $\int \sqrt{e^{3x} - e^{2x}} dx =$

- A. $\frac{2}{3}(e^x - 1)^{\frac{3}{2}} + C$
- B. $\frac{2}{3}(e^{3x} - e^{2x})^{\frac{3}{2}} + C$
- C. $\sqrt{\frac{3e^x}{2} - 1} + C$
- D. $\frac{2}{3}e^x(e^x - 1)^{\frac{3}{2}} + C$
- E. $\frac{1}{3}e^{3x} - \frac{1}{2}e^{2x} + C$
- F. None of these

27. Using integration by parts once, the integral $\int (\ln x)^{2023} dx$ becomes

- A. $2023 \frac{(\ln x)^{2022}}{x} + C$
- B. $x(\ln x)^{2023} - \int (\ln x)^{2022} dx$
- C. $2023 \int (\ln x)^{2022} dx$
- D. $x(\ln x)^{2023} - 2023 \int (\ln x)^{2022} dx$
- E. $\frac{1}{2024}(\ln x)^{2024} + C$
- F. None of these

28. Which of these is the volume of the solid formed by revolving the region bounded by the graphs $y = x$ and $y = \sqrt{x}$ about the y-axis?
- $\frac{2\pi}{15}$
 - $\frac{\pi}{6}$
 - $\frac{\pi}{2}$
 - $\frac{\pi}{15}$
 - None of these
29. $\lim_{x \rightarrow \infty} \frac{e^x}{\ln x} =$
- 0
 - ∞
 - 1
 - e
 - $-\infty$
 - None of these
30. The area of the region bounded by $x^2 - 4y + 4 = 0$ and $x - 2y + 6 = 0$ is
- 12
 - 9
 - 7
 - 5
 - 3
 - None of these
31. $\int_0^1 \frac{x}{e^{x^2}} dx$
- $\frac{1-e}{2e}$
 - $\frac{e-1}{2e}$
 - $\frac{1}{2e}$
 - $\frac{1+e}{2e}$
 - $\frac{2e}{e+1}$
 - None of these

32. $\int x^2 e^{-3x} dx =$

- A. $-\frac{1}{27}e^{-3x}(9x^2 + 6x + 2) + C$
 B. $e^{-3x}\left(-\frac{x^2}{3} + \frac{2x}{9}\right) + C$
 C. $-\frac{1}{27}e^{-3x}(9x^2 - 6x + 2) + C$
 D. $-\frac{1}{27}e^{-3x}(9x^2 - 6x - 2) + C$
 E. $e^{-3x}(9x^2 + 6x + 2) + C$
 F. None of these

33. $\int x^4(1 - 2x^5)^6 dx =$

- A. $\frac{(1-2x^5)^7}{-7} + C$
 B. $\frac{(1-2x^5)^7}{7} + C$
 C. $\frac{(1-2x^5)^7}{-70} + C$
 D. $\frac{(1-2x^5)^7}{70} + C$

34. $\int \left(\frac{\sqrt{x}+1}{x}\right) dx =$

- A. $-2\sqrt{x} + \ln|x| + C$
 B. $\frac{1}{\sqrt{x}} + \ln|x| + C$
 C. $-\frac{1}{\sqrt{x}} + \ln|x| + C$
 D. $2\sqrt{x} + \ln|x| + C$

35. $\int \frac{x-2}{x^2-4x+7} dx =$

- A. $2 \ln|x^2 - 4x + 7| + C$
 B. $\frac{1}{2} \ln|x^2 - 4x + 7| + C$
 C. $\ln|x^2 - 4x + 7| + C$
 D. None of these

36. $\int \frac{\cos(\frac{1}{x})}{x^2} dx =$

- $-\sin\left(\frac{1}{x}\right) + C$
- $\sin\left(\frac{1}{x}\right) + C$
- $\cos\left(\frac{1}{x}\right) + C$
- $-\cos\left(\frac{1}{x}\right) + C$

37. $\int \tan^2 x \cos^3 x dx =$

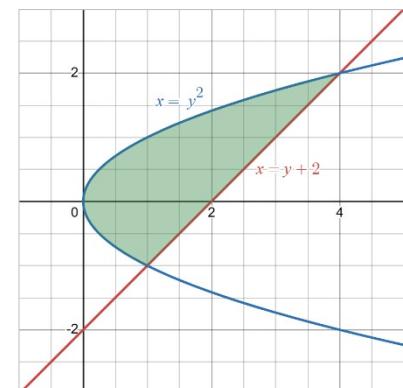
- $\frac{(\sin x)^3}{3} + C$
- $-\frac{(\sin x)^3}{3} + C$
- $-\frac{(\cos x)^3}{3} + C$
- $\frac{(\cos x)^3}{3} + C$

38. $\int x^2 \ln x dx$

- $\frac{x^3 \ln x}{3} - \frac{x^3}{3} + C$
- $\frac{x^3 \ln x}{3} - \frac{x^3}{9} + C$
- $\frac{x^4 \ln x}{4} - \frac{x^3}{4} + C$
- $\frac{x^4 \ln x}{4} - \frac{x^3}{16} + C$

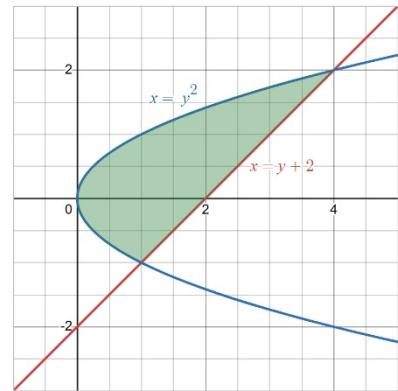
39. Set up an integral to evaluate the area of the region bounded by $x = y^2$ and $x = y + 2$.

- $\int_{-2}^1 y + 2 - y^2 dy$
- $\int_{-1}^2 -y^2 - y - 2 dy$
- $\int_1^4 \sqrt{x} - x + 2 dx$
- $\int_{-1}^2 y + 2 - y^2 dy$
- $\int_0^4 \sqrt{x} - x + 2 dx$
- $\int_{-2}^1 y + 2 + y^2 dy$



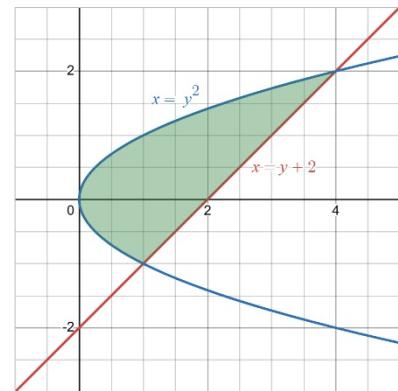
40. Set up an integral that represents the volume of the solid generated by revolving the region bounded by $x = y^2$ and $x = y + 2$ about the line $y = 3$.

- A. $\int_{-1}^2 (3 - y)(y + 2 - y^2) dy$
- B. $\int_{-1}^2 2\pi(3 - y)(y + 2 - y^2) dy$
- C. $\int_{-2}^1 2\pi(3 - y)(y^2 - y - 2) dy$
- D. $\int_1^4 (3 - \sqrt{x})^2 - (5 - x)^2 dx$
- E. $\int_{-1}^2 2\pi(3 + y)(y + 2 - y^2) dy$
- F. $\pi \int_1^4 (3 - \sqrt{x})^2 - (5 - x)^2 dx$



41. Set up an integral that represents the volume of the solid generated by revolving the region bounded by $x = y^2$ and $x = y + 2$ about the line $x = 5$.

- A. $\int_{-1}^2 (5 - y^2)^2 - (3 - y)^2 dy$
- B. $\pi \int_{-2}^1 (5 - y^2)^2 - (3 - y)^2 dy$
- C. $2\pi \int_1^4 (5 - x)(\sqrt{x} - x - 2) dx$
- D. $\pi \int_{-2}^1 (5 - y^2)^2 - (5 - y)^2 dy$
- E. $\int_1^4 (5 - x)(\sqrt{x} - x - 2) dx$
- F. $\pi \int_{-1}^2 (5 - y^2)^2 - (3 - y)^2 dy$



42. Evaluate the integral $\int 9x \sin(3x) dx$.

- A. $-3x \cos(3x) + \sin(3x) + C$
- B. $-3 \cos(3x) - \sin(3x) + C$
- C. $-3x^2 \cos(3x) + C$
- D. $3x \cos(3x) + C$
- E. $-3x \cos(3x) + 9 \sin(3x) + C$
- F. $3x \cos(3x) + \sin(3x) + C$

43. Evaluate the integral $\int \sin^{-1} x dx$.

- A. $x \sin^{-1} x + 2\sqrt{1 - x^2} + C$
- B. $x \sin^{-1} x - \sqrt{1 - x^2} + C$
- C. $x \sin^{-1} x + \sqrt{1 - x^2} + C$
- D. $\sqrt{1 - x^2} + C$
- E. $x \sin^{-1} x - \frac{1}{2}\sqrt{1 - x^2} + C$

44. Evaluate the integral $\int \frac{3^x}{1+9^x} dx$.

- A. $\ln|1 + 9^x| + C$
- B. $\tan^{-1} 3^x + C$
- C. $\ln|1 + 3^x| + C$
- D. $\frac{1}{\ln 3} \tan^{-1} 3^x + C$
- E. $\frac{1}{\ln 3} \ln|1 + 3^x| + C$
- F. $3^x + \ln|1 + 9^x| + C$

45. Evaluate the integral $\int \csc^4 x dx$.

- A. $-\frac{1}{3} \cot^3 x - \cot x + C$
- B. $\frac{1}{3} \cot^3 x + \cot x + C$
- C. $-\cot^3 x - \cot x + C$
- D. $\frac{1}{5} \csc^5 x + C$
- E. $\frac{1}{3} \cot^3 x + C$
- F. $-\cot^3 x + x + C$

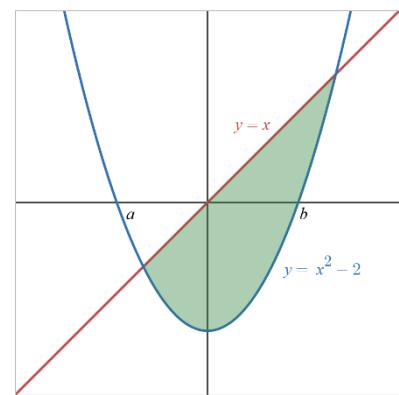
46. Evaluate the integral $\int \frac{1}{x \ln(x^5)} dx$.

- A. $\ln|\ln(x^5)| + C$
- B. $\frac{1}{5} \ln|x \ln(x^5)| + C$
- C. $\frac{2}{5} \ln|\ln(x^5)| + C$
- D. $\frac{1}{5} \ln|\ln x| + C$
- E. $5 \ln|\ln(x^5)| + C$
- F. $\frac{5}{4} \ln|\ln(x^5)| + C$

Consider the region \mathcal{R} enclosed by the functions $y = x^2 - 2$ and $y = x$ (shaded on the right), where a and b are the x-coordinates of the intersection points between the two curves. Questions 47-50 are related to this region \mathcal{R} .

47. The values of a and b are

- A. $a = -2, b = 1$
- B. $a = 2, b = -1$
- C. $a = -1, b = 2$
- D. $a = 1, b = -2$
- E. $a = 3, b = 0$
- F. $a = 0, b = 3$



48. The area of the region \mathcal{R} is

- A. $\frac{1}{2}$
- B. $\frac{1}{6}$
- C. $-\frac{1}{6}$
- D. $\frac{9}{2}$
- E. $-\frac{3}{2}$
- F. $\frac{3}{2}$

49. An integral that computes the volume of the solid of revolving the shaded region \mathcal{R} about the line $y = 7$ is

- A. $\pi \int_a^b (7 - x)^2 - (7 - x^2 + 2)^2 dx$
- B. $\pi \int_a^b (7 - x^2 + 2)^2 - (7 - x)^2 dx$
- C. $\pi \int_a^b (7 - y)^2 - (9 - y)^2 dy$
- D. $2\pi \int_a^b (7 - y)(y + y^2 - 2) dy$
- E. $2\pi \int_a^b (7 - x)(x - x^2 + 2) dx$
- F. $\pi \int_a^b (7 - x)^2 - (5 - x^2)^2 dx$

50. An integral that computes the volume of the solid of revolving the shaded region \mathcal{R} about the line $x = 7$ is

- A. $\pi \int_a^b (7 - x)^2 - (7 - x^2 + 2)^2 dx$
- B. $\pi \int_a^b (7 - x^2 + 2)^2 - (7 - x)^2 dx$
- C. $\pi \int_a^b (7 - y)^2 - (9 - y)^2 dy$
- D. $2\pi \int_a^b (7 - y)(y + y^2 - 2) dy$
- E. $2\pi \int_a^b (7 - x)(x - x^2 + 2) dx$
- F. $\pi \int_a^b (7 - x)^2 - (5 - x^2)^2 dx$

51. $\int 5x^9 e^{x^5} dx =$

- A. $\frac{5x^7 e^{x^5}}{7}$
- B. $e^{x^5} + C$
- C. $x^5 e^{x^5} - e^{x^5} + C$
- D. $x^6 e^{x^5} + C$
- E. $x^{5x^6} e^{x^5} + C$
- F. $xe^x - e^x + C$

52. Evaluate $\lim_{x \rightarrow \infty} \left(1 + \frac{4}{x^2}\right)^{x^2}$.

- A. 1
- B. 3
- C. 4
- D. $-\infty$
- E. 0
- F. None of the above

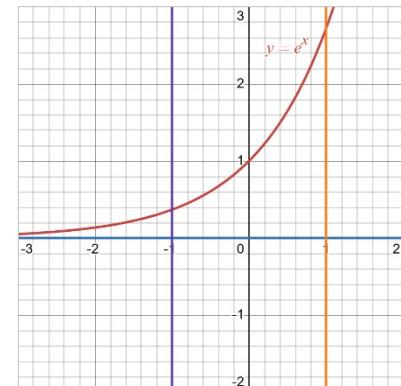
53. Evaluate $\int_{-1}^1 \frac{xe^{x^2}+1}{1+x^2} dx$.

- A. 1
- B. $-\pi$
- C. π
- D. $\frac{\pi}{2}$
- E. 0
- F. None of the above

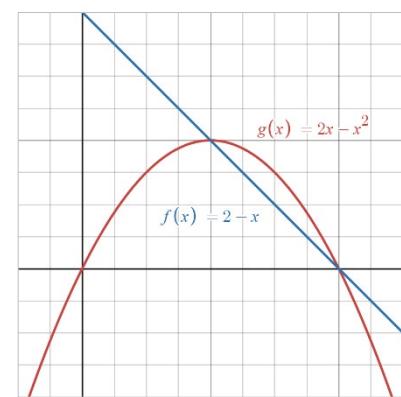
Written Questions.

1. Consider the region \mathfrak{R} enclosed by the curves $y = e^x$, $y = 0$, $x = -1$ and $x = 1$.

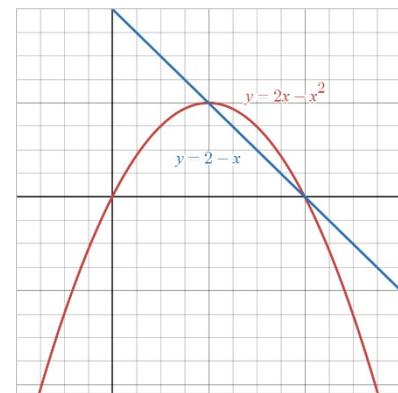
- a) Find the area of the region \mathfrak{R} .



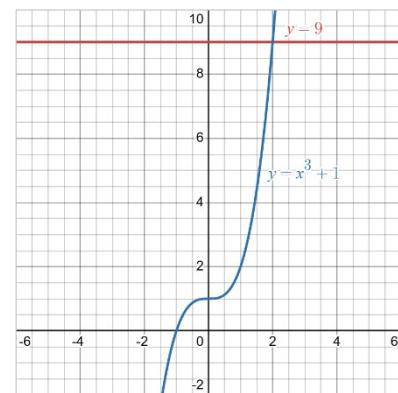
- b) Set up but do not evaluate the integral that computes the volume of the solid by rotating the region \mathfrak{R} about the line $y = -1$.
- c) Set up but do not evaluate the integral that computes the volume of the solid by rotating the region \mathfrak{R} about the line $x = -2$.
2. Compute the area bounded by the graph of $f(x) = 2 - x$ and $g(x) = 2x - x^2$.



3. Write down integral which will give you the volume of the solid generated when the region bounded by the graphs of $y = 2 - x$ and $y = 2x - x^2$ is revolved around the line $y = -2$.

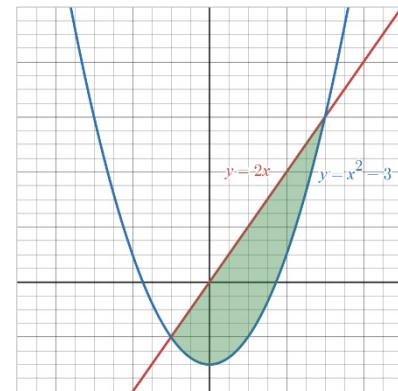


4. By using the cylindrical shells method, write down integral which will give you the volume of the solid generated when the region bounded by the graphs of $y = x^3 + 1$, $y = 9$ and $x = 0$ is revolved around the y-axis.



5. Consider the region R enclosed by the curves $y = x^2 - 3$ and $y = 2x$.

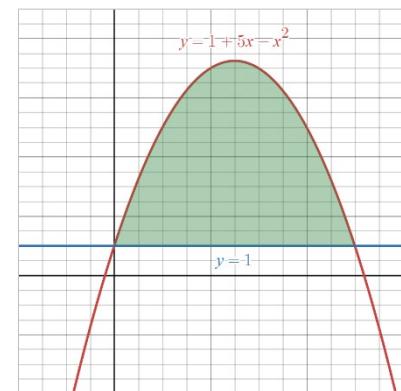
- a) Find the area of the region R.



- b) Write down the integral that computes the volume of the solid generated by the rotation of the region R about the line $y = 7$.

6. Consider the region bounded by the functions $y = 1$ and $y = 1 + 5x - x^2$.

a) Find the area of the shaded region.



- b) Set up but do not evaluate the integral that computes the volume of the solid by rotating the region about the line $y = 1$.
- c) Set up but do not evaluate the integral that computes the volume of the solid by rotating the region about the y-axis.
- d) Set up but do not evaluate the integral that computes the volume of the solid by rotating the region about the line $x = -1$.

7. Let R be the region bounded by $y = 4 - x^2$ and $y = 0$. Find the volume of the solid obtained by revolving R about the line $x = 3$.

8. Find the area bounded by the graph of $y = x^2$, $y = 2 - x$ and $y = 0$.

9. Let R be the shaded region bounded by the graph of the functions $y = x$ and $y = x^2 - 2$.

a) Set up an integral to find the area of the region R.

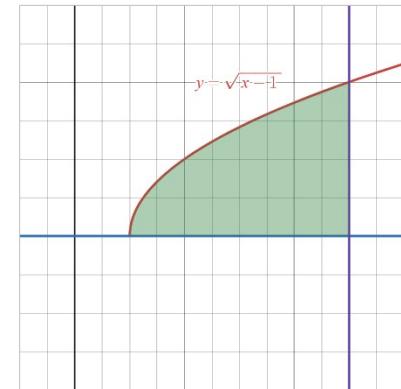
b) Set up an integral that can be used to find the volume of the solid generated by revolving R about the line $x = -3$.

c) Set up an integral that can be used to find the volume of the solid generated by revolving R about the line $y = 4$.

10. Set up but do not evaluate the integral that represents the volume of the solid generated by revolving the region

$y = \sqrt{x-1}$, $y = 0$ and $x = 5$ about the line $y = -2$.

a) Using the disk/washer method.



b) Using the cylindrical shell method.

11. Evaluate the integral $\int x \tan^{-1}(x) dx$.

12. Evaluate the integral $\int x^3 e^{x^2} dx$.

13. Evaluate the integral $\int \sin(\ln x) dx$.

14. Evaluate the indefinite integral $\int \ln(\sqrt{x}) dx$.

15. Evaluate the integral $\int x^2 \sin x dx$.

16. Evaluate the integral $\int t \cos^5(t^2) dt$.

17. Evaluate the integral $\int e^x \cos x dx$.

18. Evaluate the integral $\int (x - 2)(x + 2)^{\frac{1}{3}} dx$.

19. Find $\int_0^{\frac{\pi}{2}} \cos x \sin(\sin x) dx$.

20. Find $\lim_{x \rightarrow 0^+} (\cosh x)^{\frac{1}{x}}$.

21. Find $\lim_{x \rightarrow \infty} e^x(1 - \cos(e^{-x}))$.

22. Let f be continuous on $[0, 9]$ and $\int_0^9 f(x) dx = 6$. Evaluate the definite integral $\int_{-3}^3 |x|f(x^2) dx$.

23. If $\int_0^{\frac{\pi}{2}} \cos(x^2) dx = 0.84914$, then find $\int_0^{\frac{\pi}{2}} 2x^2 \sin(x^2) dx$.