OL Academy

MATHS101

Lesson 10

Test 1 Revision

(1) Find $\lim_{x \to 2} \frac{x^3 - 3x^2 + 2x}{x - 2} =$

A. ∞

B. 2

C. $\frac{1}{2}$

D. 24

E. $\frac{1}{24}$

F. 1

$$\lim_{x \to 2} \frac{x\left[x^2 - 3x + 2\right]}{x - 2}$$

$ax^2 \pm bx \pm c = 0$ mode 5 3

$x_1 = 2, \; x_2 = 1$

$(x-2)(x-1) = 0$

$$\lim_{x \to 2} \frac{x\left[\cancel{(x-2)}(x-1)\right]}{\cancel{x-2}}$$

$$\lim_{x \to 2} x(x-1) \Rightarrow (2)(2-1) = (2)(1) = 2$$

(2) $\lim_{x \to -3} \dfrac{x^2 + 2x - 3}{x^2 + 5x + 6} =$

A. -4

B. $\dfrac{3}{4}$

C. 6

D. $\dfrac{4}{3}$

E. 0

F. 4

$$\lim_{x \to -3} \frac{(x-1)\cancel{(x+3)}}{(x+2)\cancel{(x+3)}} \Rightarrow \lim_{x \to -3} \frac{x-1}{x+2} = \frac{(-3)-1}{-3+2} = \frac{-4}{-1}$$

$$= 4$$

(3) $\lim_{x\to 5} \frac{x-5}{\sqrt{x+4}-3} =$ $\times \frac{\sqrt{x+4}+3}{\sqrt{x+4}+3}$

A. $\frac{1}{6}$

B. 6

C. 0

D. -6

E. 7

F. 12

$(\sqrt{x+4}-3) \times (\sqrt{x+4}+3)$

$(x+4+3\cancel{\sqrt{x+4}}-3\cancel{\sqrt{x+4}}-9)$

$= x+4-9 = x-5$

$$\lim_{x\to 5} \frac{(\cancel{x-5})\times(\sqrt{x+4}+3)}{\cancel{x-5}}$$

$$\lim_{x\to 5} \sqrt{x+4}+3 \Rightarrow \sqrt{5+4}+3 \Rightarrow \sqrt{9}+3 = 3+3 = 6$$

(4) $\lim_{x \to 2} \dfrac{\frac{1}{x} - \frac{1}{2}}{x - 2} =$

A. $\dfrac{-1}{4}$ B. $\dfrac{1}{8}$ C. 2

D. $\dfrac{1}{4}$ E. -4 F. 4

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{ad}{bc}$$

$$\lim_{x \to 2} \frac{\frac{1}{x} - \frac{1}{2}}{x - 2} = \lim_{x \to 2} \frac{\frac{2-x}{2x}}{\frac{x-2}{1}} = \lim_{x \to 2} \frac{2-x}{(x-2)\,2x}$$

$$= \lim_{x \to 2} \frac{-\cancel{(x-2)}}{2x\,\cancel{(x-2)}} = \lim_{x \to 2} \frac{-1}{2x} = -\frac{1}{4}$$

(5) What is the largest interval where the function $f(x) = \underbrace{\sqrt{3x+18}}$ continuous?

A. $(6, \infty)$ B. $(-\infty, 6]$ C. $(\infty, -6)$

D. $[-18, \infty)$ E. $[6, \infty)$ F. $[-6, \infty)$

$3x + 18 \geq 0$

$\frac{3x}{3} \geq \frac{-18}{3}$

$x \geq -6 \rightarrow [-6, \infty)$

(6) If $\underbrace{\frac{5-x^2}{x+1}}$ $\leqslant f(x) \leqslant$ $\underbrace{\frac{25+x^2}{5+x^2}}$, then $\lim_{x\to 0} f(x) =$

$g(x) \leq f(x) \leq h(x)$

A. 0

B. -2

C. 5

$\lim g(x) \leq \lim f(x) \leq \lim h(x)$

D. -3

E. -5

F. None of the above

$$\lim_{x\to 0} \frac{5-x^2}{x-1} = 5 \quad , \quad \lim_{x\to 0} \frac{25+x^2}{5+x^2} = \frac{25}{5} = 5$$

$$\lim_{x\to 0} f(x) = 5$$

(7) If

$$f(x) = \begin{cases} x^4 + 3a + 5, & x \leqslant 1 \\ ax, & x > 1 \end{cases}$$

$$\lim_{x \to 1^-} = \lim_{x \to 1^+}$$

is a continuous function, then $a =$

A. 6 B. 3 C. 1

D. -3 E. -1 F. -6

$$\lim_{x \to 1^-} x^4 + 3a + 5 = (1)^4 + 3a + 5 = 3a + 6$$

$$\lim_{x \to 1^+} ax = a \Rightarrow a = 3a + 6 \Rightarrow 2a = -6$$

$$\Rightarrow a = -3$$

(8) The points of discontinuity of $f(x) = \dfrac{x^2-4}{x^3-x}$ are

A. $-1, 0, 1$ only
B. $1, 0$ only
C. $2, -2, 0, -1, 1$ only
D. $2, -2, 1, -1$ only
E. $-1, 1$ only
F. $2, -2$ only

$* \quad \sqrt{x^2} = |x|$
$= \pm x$

$x^3 - x = 0$

$x(x^2-1) = 0$

$x = 0$, $x^2 - 1 = 0$

$\sqrt{x^2} = \sqrt{1}$

$x = \pm 1$

$\Rightarrow x = -1, 0, 1$ only

(9) $\lim_{x\to-\infty} \dfrac{\sqrt{9x^2+1}}{2x-7} =$

$* \lim_{x\to\pm\infty} \frac{1}{x^n} = 0 \quad , n = 1, 2, 3, \ldots \ \& \ \frac{1}{2}, \frac{1}{3}, \ldots$

A. $\dfrac{-3}{2}$ B. $\dfrac{3}{2}$ C. $\dfrac{-\sqrt{3}}{2}$

D. ∞ E. $-\infty$ F. 0

$$\lim_{x\to-\infty} \frac{\sqrt{9x^2+1}}{2x-7} \longrightarrow \lim_{x\to-\infty} \sqrt{x^2\cdot(9+1/x^2)} = \underbrace{\sqrt{x^2}}_{|x|} \sqrt{9+1/x^2}$$

$$\lim_{x\to-\infty} \frac{\underbrace{|x|}_{-x} \sqrt{9+1/x^2}}{2x-7} \Rightarrow \lim_{x\to-\infty} \frac{\left(-x\sqrt{9+1/x^2}\right)/x}{(2x-7)/x}$$

$$\lim_{x\to-\infty} \frac{-\sqrt{9+\cancel{1/x^2}_{\,0}}}{2-\cancel{7/x}_{\,0}} \Rightarrow \frac{-\sqrt{9}}{2} = -\frac{3}{2}$$

$\lim_{x\to1^+} |x| = x = 1$

$\lim_{x\to1^-} |x| = -x = -1$

D.N.E

(10) $\lim_{x\to\infty} \dfrac{x^3+3x+6}{x^5+2x^2+9} =$

A. 1

B. $\dfrac{5}{3}$

C. $\dfrac{3}{5}$

D. ∞

E. $-\infty$

F. 0

$$\lim_{x\to\infty} \frac{(x^3+3x+6)\div x^5}{(x^5+2x^2+9)\div x^5} \Rightarrow \lim_{x\to\infty} \frac{\frac{1}{x^2}+\frac{3}{x^4}+\frac{6}{x^5}}{1+\frac{2}{x^3}+\frac{9}{x^5}}$$

$$= \frac{0}{1} = 0$$

(11) $\lim_{x\to 5^+} \dfrac{4+x}{5-x} =$

A. $\dfrac{4}{5}$

B. 1

C. 5

D. ∞

E. 0

F. $-\infty$

$\dfrac{1}{\infty} = 0$

$\dfrac{1}{0.000000001} = \infty$

$\lim_{x\to 5^+} \dfrac{4+x}{5-x} \to \dfrac{4+5}{5-(5.000001)} \to \dfrac{9}{-0.0000001}$

$= -\infty$

(12) $\lim_{x\to 1^-} \dfrac{x-1}{\sqrt{(x-1)^2}} =$

A. 1

B. $-\infty$

C. 2

D. 0

E. ∞

F. -1

$$\sqrt{x^2} = |x| = \pm x$$

$$\lim_{x\to a^+} x = a$$

$$\lim_{x\to a^-} -x = -a$$

$$\lim_{x\to a} \text{D.N.E}$$

$$\lim_{x\to a^-} |x| \neq \lim_{x\to a^+} |x|$$

$$\lim_{x\to 1^-} \frac{x-1}{|x-1|}$$

$$\lim_{x\to 1^-} \frac{\cancel{x-1}}{-(\cancel{x-1})} \Rightarrow \lim_{x\to 1^-} \frac{1}{-1} = -1$$

(13) $\lim_{x\to 2} \frac{|x-2|}{x^2+x-6} =$

A. 1

B. $-\infty$

C. 2

D. 0

E. Does not exist

F. ∞

$$\lim_{x\to 2} \frac{|x-2|}{(x-2)(x+3)} \Rightarrow \lim_{x\to 2^+} \frac{+(\cancel{x-2})}{(\cancel{x-2})(x+3)} = \lim_{x\to 2^+} \frac{1}{x+3} = \frac{1}{5}$$

$$\Rightarrow \lim_{x\to 2^-} \frac{-(\cancel{x-2})}{(\cancel{x-2})(x+3)} = \lim_{x\to 2^-} \frac{-1}{x+3} = \frac{-1}{5}$$

$$\lim_{x\to 2^+} f(x) \neq \lim_{x\to 2^-} f(x) \Rightarrow \lim_{x\to 2} f(x) = \text{D.N.E}$$

(14) If $\lim_{x \to 0^+} f(x) = 3$ and $\lim_{x \to 0^-} f(x) = -3$, then $\lim_{x \to 0} f(x) =$

A. -3

B. ∞

C. Does not exist

D. $-\infty$

E. 3

F. 0

(15) $\lim_{x\to 3^-} \frac{2(x-3)}{|x-3|} =$

A. 2

B. 0

C. 1

D. -1

E. Does not exist

F. -2

$$\lim_{x\to 3^-} \frac{2\cancel{(x-3)}}{-\cancel{(x-3)}} = \lim_{x\to 3^-} \frac{2}{-1} = -2$$

(16) $\lim_{x\to 0^-} \dfrac{x-|x|}{x} =$

A. -2

B. 2

C. 0

D. 1

E. -1

F. Does not exist

$$\lim_{x\to 0^-} \frac{x-(-x)}{x} \Rightarrow \lim_{x\to 0^-} \frac{x+x}{x} \Rightarrow \lim_{x\to 0^-} \frac{2\cancel{x}}{\cancel{x}} = 2$$

(17) If

$$f(x) = \begin{cases} 5 + x^2, & x < -3 \\ 1 - 3x, & x \geqslant -3 \end{cases}$$

$5 > -3 \Rightarrow x \geqslant -3$

, then $\lim_{x \to 5} f(x) =$

A. 5

B. 1

C. 21

D. 14

E. -14

F. Does not exist

$$\left(\lim_{x \to 5^+} 1 - 3x = \lim_{x \to 5^-} 5 + x^2 \right)$$

$, \; 5 > -3$

$$-14 \neq 30$$

$$\lim_{x \to 5} 1 - 3x = 1 - 3(5) = 1 - 15 = -14$$

(18) If $\lim_{x\to 2} \frac{f(x)-1}{x^3+2} = 1$, then $\lim_{x\to 2} f(x) =$

A. $\frac{1}{9}$

B. 1

C. -9

D. 11

E. $\frac{-1}{9}$

F. Does not exist

① $(3x = 1) \div 3$

$x = \frac{1}{3}$

② $\left(\frac{x}{3} = 1\right) \times 3$

$x = 3$

$\frac{x-1}{10} = 1$

$\lim_{x\to 2} f(x) = f(2)$

$\lim_{x\to 2} \frac{f(x)-1}{x^3+2} = 1 \implies \frac{f(2)-1}{(2)^3+2} = 1 \implies \left(\frac{f(2)-1}{10} = 1\right) \times 10$

$\implies f(2) - 1 = 10 \implies f(2) = 11$

(19) If $\lim_{x \to 10} \dfrac{f(x) - 1}{x - 10} = 5$, then $\lim_{x \to 10} f(x) =$

A. -10 B. 1 C. 10

D. -1 E. 5 F. Does not exist

$$\lim_{x \to 10} \frac{f(x) - 1}{x - 10} = 5 \implies \frac{f(10) - 1}{\cancel{\underbrace{(10-10)}_{=0}}} = 5$$

(20) If $\lim_{x\to 5} f(x) = -3$, then $\lim_{x\to 5}(f(x)+4)^{2017}$

A. 4^{2017}

B. ∞

C. Does not exist

D. $-\infty$

E. 2017

F. 1

$$\lim_{x\to 5} f(x) = -3 \iff f(5) = -3$$

$$\lim_{x\to 5} (f(x)+4)^{2017} \Rightarrow (f(5)+4)^{2017} \Rightarrow (-3+4)^{2017}$$

$$\Rightarrow (1)^{2017} = 1$$

(3) If $f(x) = \frac{4}{x+5}$, then $f'(0) =$

(A) $-\frac{4}{5}$ (B) $\frac{4}{25}$ (C) $\frac{4}{5}$

(D) $-\frac{4}{25}$ (E) 0 (F) $-\frac{1}{25}$

$$\left(\frac{f}{g}\right)' = \frac{f'g - g'f}{g^2}$$

$$\frac{d}{dx} c = 0$$

$$\frac{d}{dx} x^n = n x^{n-1}$$

$$x^0 = 1$$

$$f'(x) = \frac{0(x-5) - ((1)x^{1-1} + 0)(4)}{(x+5)^2}$$

$$f'(x) = \frac{0 - 4}{(x+5)^2} = \frac{-4}{(x+5)^2}$$

$$f'(0) = \frac{-4}{((0)+5)^2} = \frac{-4}{5^2} = \frac{-4}{25}$$

(b) **(5 points)** Find the equation of the normal line to the curve $y = x^4 + 2e^x$ at the point $(0,2)$.

$x_1 \quad y_1$

Tangent line $\Rightarrow y - y_1 = m(x - x_1)$

normal line $\Rightarrow y - y_1 = -\frac{1}{m}(x - x_1)$

$m = y' = (4)x^{4-1} + 2e^x$

$= 4x^3 + 2e^x$

when $x = 0$

$\frac{d}{dx} e^x = e^x$

$\boxed{m = 2 \rightarrow -\frac{1}{m} = -\frac{1}{2}}$

$m = \cancel{4(0)^3} + 2e^0 = 2(1) = 2$

Tangent line $\Rightarrow y - 2 = 2(x - 0) \Rightarrow y - 2 = 2x \Rightarrow y = 2x + 2$

Normal line $\Rightarrow y - 2 = -\frac{1}{2}(x - 0) \Rightarrow y - 2 = -\frac{1}{2}x \Rightarrow y = -\frac{1}{2}x + 2$

(c) **(7 points)** Find $f'(x)$ if

$$f(x) = \frac{x^2 e^x}{x^3 + x^{3/2}}$$

(Do not simplify your answer).

$$\left(\frac{f}{g}\right)' = \frac{f'g - g'f}{g^2}$$

$$~~(f.g)' = f'g + g'f~~$$

$$f'(x) = \frac{\left[\left[(2x e^x) + (x^2 e^x)\right](x^3 + x^{3/2})\right] - \left[(3x^2 + \tfrac{3}{2}x^{1/2})(x^2 e^x)\right]}{(x^3 + x^{3/2})^2}$$

find $f''(0)$ and $f''(1)$ if

$$f(x) = x^3 e^x + 2x^{10}$$

① $f'(x) = (3x^2 e^x + e^x x^3) + (2)(10)x^9$

$= 3x^2 e^x + x^3 e^x + 20x^9$

② $f''(x) = ((2)(3)x e^x) + (3x^2 e^x) + (3x^2 e^x + x^3 e^x) + (20)(9)x^8$

$= 6xe^x + 3x^2 e^x + 3x^2 e^x + x^3 e^x + 180x^8$

$= 6xe^x + 6x^2 e^x + x^3 e^x + 180x^8$

$\Rightarrow$

$$f''(x) = 6xe^x + 6x^2e^x + x^3e^x + 180x^8$$

$$f''(0) = 0$$

$$f''(1) = 6(1)e^1 + 6(1)^2e^1 + (1)^3e^1 + 180(1)^8$$

$$= 6e + 6e + e + 180$$

$$= 13e + 180$$