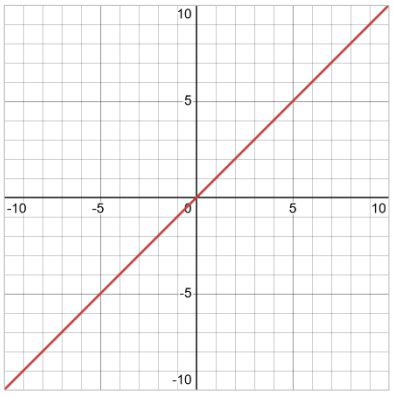
Famous Graphs

$y = x$



$$y = a(x - h) + k$$

- **Stretching and Compressing:**
  - If $|a| > 1$ ⇒ Stretch
  - If $0 < |a| < 1$ ⇒ Compress
- **Reflecting:**
  - If a is negative ⇒ Reflect about x-axis
- **Horizontal Shifts:**
  - If h is positive ⇒ Shift to the right by h units
  - If h is negative ⇒ Shift to the left by h units
- **Vertical Shifts:**
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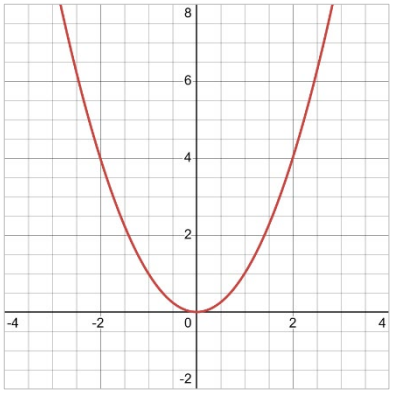
$y = x^2$



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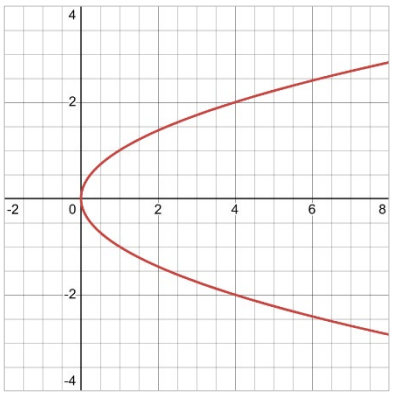
$x = y^2$



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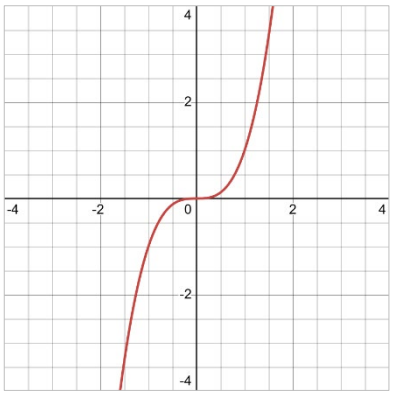
$y = x^3$



$$y = a(x-h)^3 + k$$

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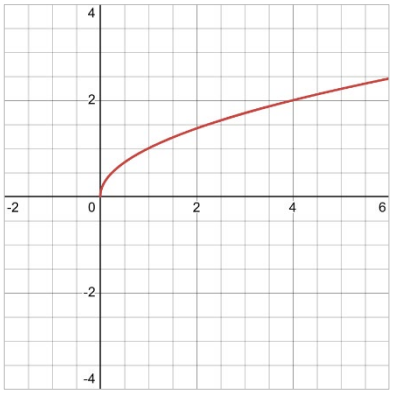
$y = \sqrt{x}$



$$y = a\sqrt{x-h} + k$$

- **Stretching and Compressing:**
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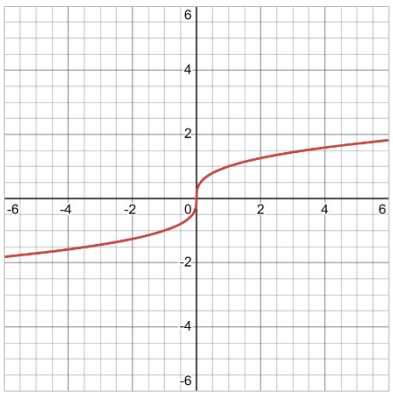
$y = \sqrt[3]{x}$



$$y = a\sqrt[3]{(x-h)} + k$$

- **Stretching and Compressing:**
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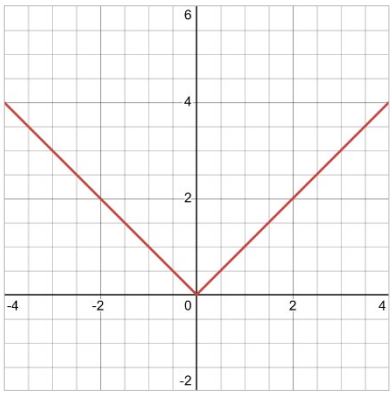
$y = |x|$



$$y = a|x - h| + k$$

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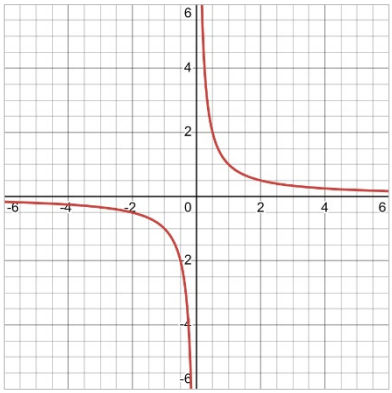
$y = 1/x$



$$x = a/(x - h) + k$$

- **Stretching and Compressing:**
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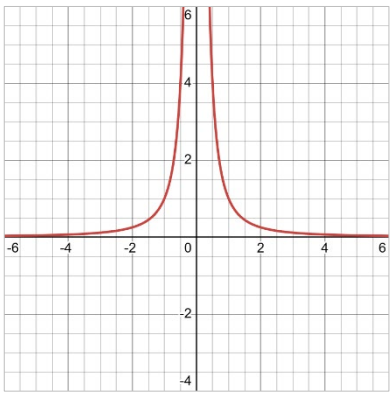
$y = 1/x^2$



$$x = a/(x - h)^2 + k$$

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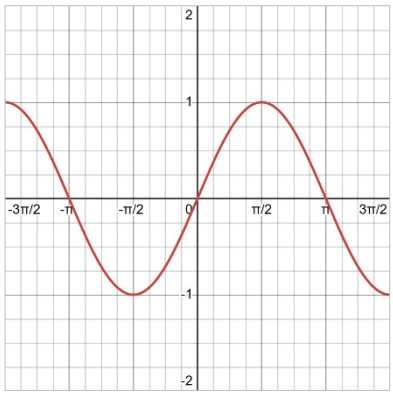
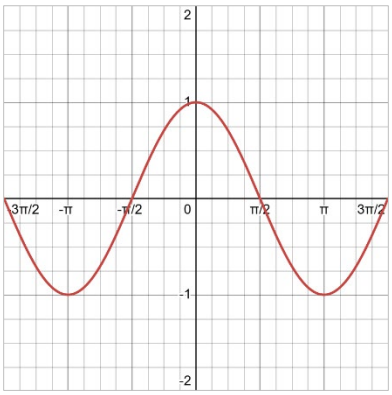
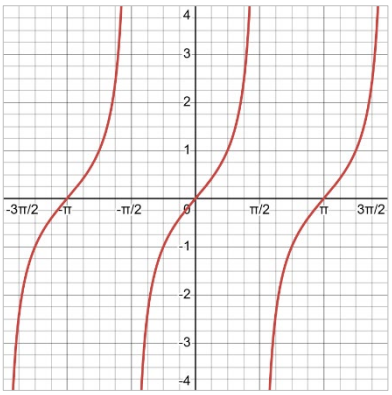
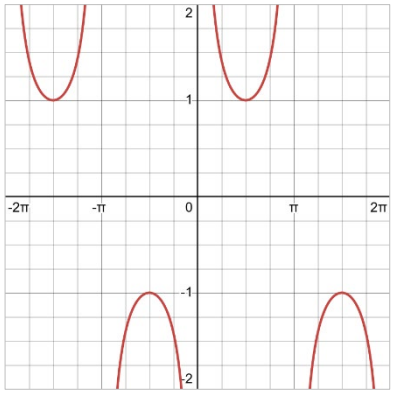
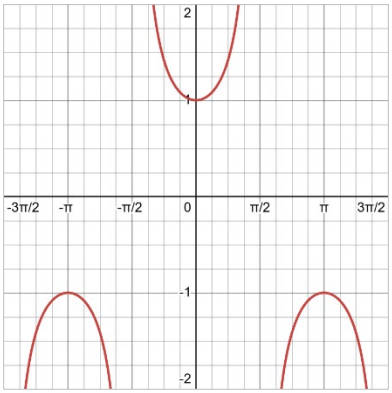
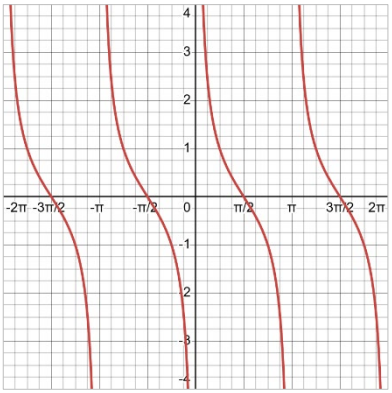
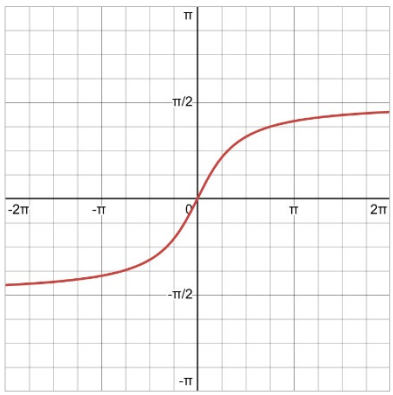
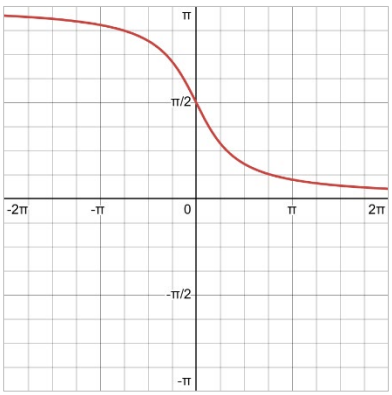
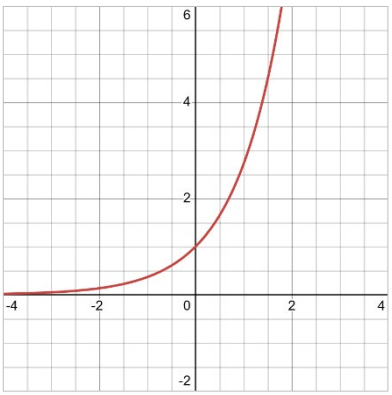
$y = \sin x$	$y = \cos x$	$y = \tan x$
		
$y = \csc x$	$y = \sec x$	$y = \cot x$
		
$y = tan^{-1} x$	$y = \cot^{-1} x$	$y = e^x$
		

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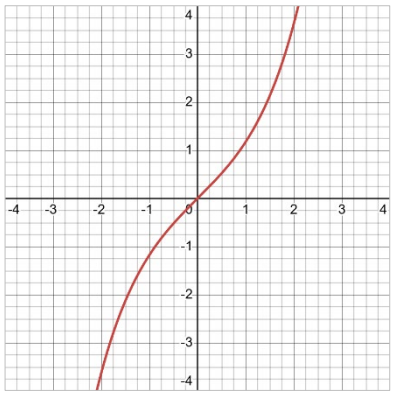
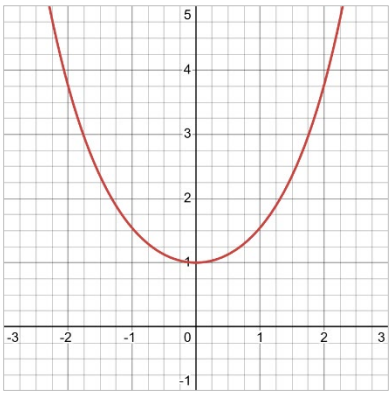
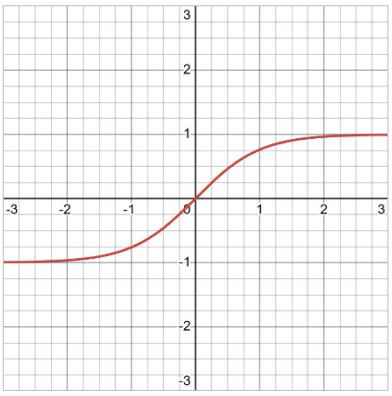
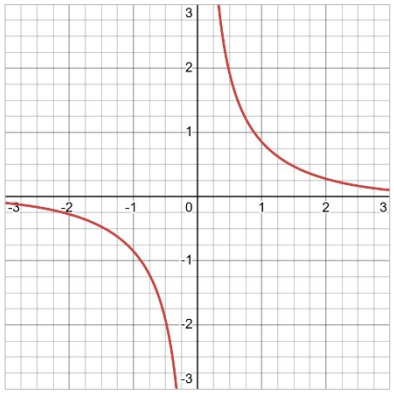
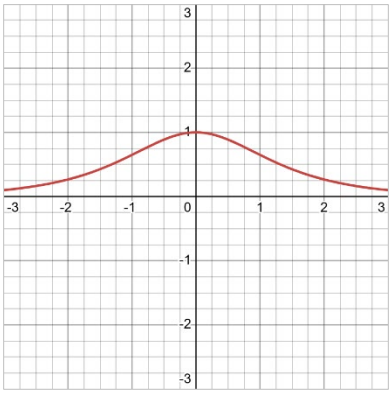
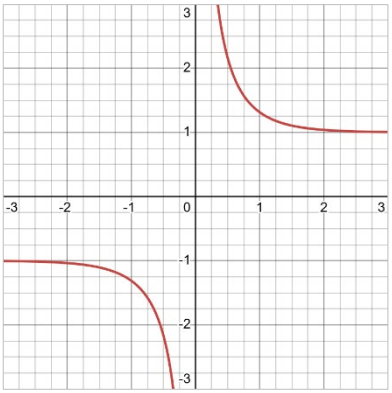
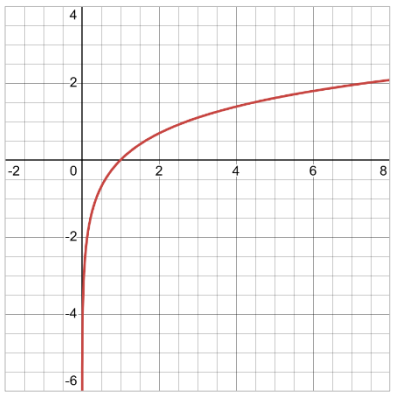
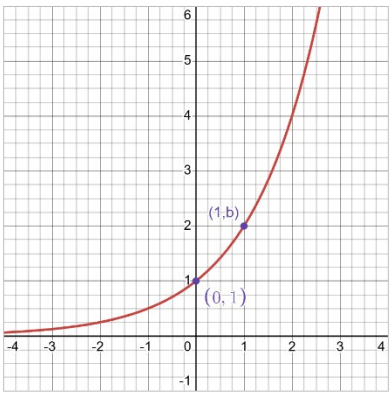
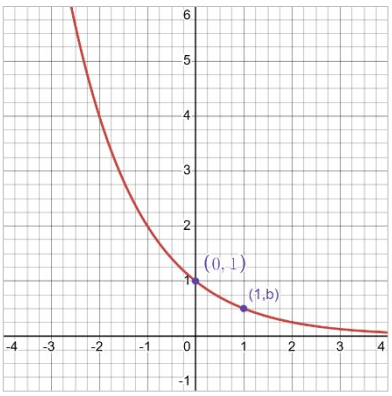
$y = \sinh x$	$y = \cosh x$	$y = \tanh x$
		
$y = \operatorname{csch} x$	$y = \operatorname{sech} x$	$y = \coth x$
		
$y = \ln x$	$y = b^x \ , b > 1$	$y = b^x \ , 0 < b < 1$
		

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How to sketch the graph of $f(x) = ax^2 + bx + c$?

1. Label a , b , and c.
2. If $a > 0$ (positive), then the parabola opens upwards (concave up).
   If $a < 0$ (negative), then the parabola opens downwards (concave down).
3. Find the vertex (h, k):
   $h = \dfrac{-b}{2a}$, $k = ah^2 + bh + c$
4. Find the x-intercept $(x, 0)$:
   $f(x) = 0 \implies ax^2 + bx + c = 0$
5. Find the y-intercept $(0, c)$:
   $y = a(0)^2 + b(0) + c = c$

➢ **Example:**

Sketch $f(x) = x^2 - 2x$.

$a = 1 , b = -2 , c = 0$

$a > 0 \Rightarrow$ concave up

- **vertex:**

$h = \dfrac{-b}{2a} = \dfrac{-(-2)}{2(1)} = \dfrac{2}{2} = 1$

$k = ah^2 + bh + c = 1^2 - 2(1)^2 = -1$

$\implies (1, -1)$

- **x-intercept:**

$f(x) = 0 \Rightarrow x^2 - 2x = 0 \Rightarrow x(x-2) = 0$

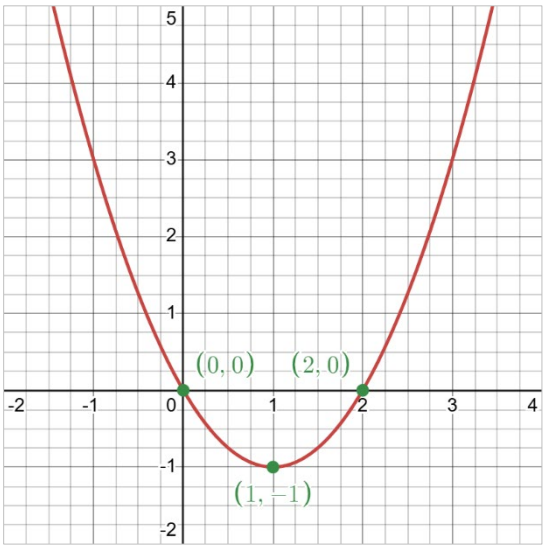
$\Rightarrow x = 0$ or $x = 2$

$x = 0 \implies f(0) = 0^2 - 2(0) = 0 \implies (0, 0)$

$x = 2 \implies f(0) = 2^2 - 2(2) = 0 \implies (2, 0)$

- **y-intercept:**

$(0, c) \Rightarrow (0, 0)$



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