OL Academy

Lesson 8

MATHS101

3.2 The Product and Quotient Rules

3.2 The Product and Quotient Rules

The Product Rule If f and g are both differentiable, then

$$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$$

$$\frac{d}{dx} f(x)\, g(x) = f'g + g'f$$

If $f(x) = xe^x$, find $f'(x)$.

$h(x) = x \quad , \quad g(x) = e^x$

$$f'(x) = \frac{d}{dx}(xe^x) \Rightarrow x e^x + e^x (1)$$

$$= xe^x + e^x$$

$$= e^x (x+1)$$

The Quotient Rule If f and g are differentiable, then

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}[f(x)] - f(x)\frac{d}{dx}[g(x)]}{[g(x)]^2}$$

$$y' = \frac{d}{dx}$$

$f(x)$

Let $y = \dfrac{x^2 + x - 2}{x^3 + 6}$. Then $y' =$

$g(x)$

$$y' = \frac{(2x+1)(x^3+6) - (3x^2)(x^2+x-2)}{(x^3+6)^2}$$

$$y' = \frac{(2x^4 + x^3 + 12x + 6) - (3x^4 + 3x^3 - 6x^2)}{(x^3+6)^2}$$

$$y' = \frac{-x^4 - 2x^3 + 6x^2 + 12x + 6}{(x^3+6)^2}$$

Table of Differentiation Formulas

$\frac{d}{dx}(c) = 0$	$\frac{d}{dx}(x^n) = nx^{n-1}$	$\frac{d}{dx}(e^x) = e^x$
$(cf)' = cf'$	$(f+g)' = f' + g'$	$(f-g)' = f' - g'$
$(fg)' = fg' + gf'$	$\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}$	

$* \ \frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$

3–30 Differentiate.

3. $y = (4x^2 + 3)(2x + 5)$

$y' = ((4)(2x))(2x+5) + (2)(4x^2+3)$

$= 8x^2 + 6 + 16x^2 + 40x$

$= 24x^2 + 40x + 6$

8. $g(x) = \left(x + 2\sqrt{x}\right) e^x$ $= \cancel{\underbrace{xe^x} + \underbrace{2e^x\sqrt{x}}}$

$$g'(x) = \left(1 + \cancel{2} \cdot \frac{1}{\cancel{2}\sqrt{x}}\right)e^x + e^x\left(x + 2\sqrt{x}\right)$$

$$= \left(1 + \frac{1}{\sqrt{x}}\right)e^x + \left(x + 2\sqrt{x}\right)e^x$$

① $e^x + \dfrac{e^x}{\sqrt{x}} + xe^x + 2e^x\sqrt{x}$

② $e^x\left[1 + \dfrac{1}{\sqrt{x}} + x + 2\sqrt{x}\right]$

31–34 Find $f'(x)$ and $f''(x)$.

31. $f(x) = x^2 e^x$

$f'(x) = (2x)e^x + e^x(x^2) \Rightarrow 2xe^x + x^2 e^x = \underbrace{e^x(2x + x^2)}$

$f''(x) = e^x(2x + x^2) + (2 + 2x)e^x$

$= e^x[2x + x^2 + 2 + 2x] = e^x[x^2 + 4x + 2]$

43. If $f(x) = x^2/(1+x)$, find $f''(1)$.

$$\left(\frac{f}{g}\right)' = \frac{f'g - g'f}{(g)^2}$$

$$f'(x) = \frac{(2x)(1+x) - (1)(x^2)}{(1+x)^2} = \frac{x^2+2x}{(1+x)^2} \to \underbrace{1+2x+x^2}$$

$$f''(x) = \frac{(2x+2)(x^2+2x+1) - (2x+2)(x^2+2x)}{[(1+x)^2]^2}$$

$$= \frac{(2x+2)[(x^2+2x+1) - (x^2+2x)]}{(1+x)^4} = \frac{2(1+\cancel{x})[(x^2+2x+1) - (x^2+2x)]}{(1+x)^{\cancel{4}\,3}}$$

$$= \frac{2(\cancel{x^2}+\cancel{2x}+1-\cancel{x^2}-\cancel{2x})}{(1+x)^3} = \frac{2}{(1+x)^3}, \quad f''(1) = \frac{2}{(1+1)^3} = \frac{2}{8} = \frac{1}{4}$$

37–38 Find equations of the tangent line and normal line to the given curve at the specified point.

eq tangent line $\Rightarrow y - y_1 = m(x - x_1)$

eq normal line $\Rightarrow y - y_1 = -\frac{1}{m}(x - x_1)$

$m \Rightarrow y'$: slope

38. $y = x + xe^x$, $\overset{x_1}{(0}, \overset{y_1}{0)}$

① $y' = 1 + e^x + e^x x = m$

② $m = 1 + e^x(1 + x)$ at $x = 0 \rightarrow m = 1 + e^0(1 + 0) = 1 + 1 = 2$

③ $y - 0 = 2(x - 0) \Rightarrow y = 2x$

④ $m = 2 \rightarrow m = -\frac{1}{2}$

⑤ $y - 0 = -\frac{1}{2}(x - 0) \Rightarrow y = -\frac{1}{2}x$

45. Suppose that $f(5) = 1$, $f'(5) = 6$, $g(5) = -3$, and $g'(5) = 2$. Find the following values.

(a) $(fg)'(5)$

$f'(5)\,g(5) + g'(5)\,f(5)$

$(6)(-3) + (2)(1)$

$= -18 + 2$

$= -16$

(b) $(f/g)'(5)$

$$\frac{f'(5)\,g(5) - g'(5)\,f(5)}{[g(5)]^2}$$

$$\frac{(6)(-3) - (2)(1)}{(-3)^2}$$

$$= \frac{-18-2}{9} = \frac{-20}{9}$$

(c) $(g/f)'(5)$

$$\frac{g'(5)\,f(5) - f'(5)\,g(5)}{[f(5)]^2}$$

$$\frac{(2)(1) - (6)(-3)}{(1)^2}$$

$$= \frac{2+18}{1} = 20$$