OL Academy

MATHS101

Lesson 7

2.8 The Derivative as a Function

3.1 Derivative of Polynomials and Exponential Functions

Slope → Point $a = (\overset{x_1}{4}, \overset{y_1}{6})$, Point $b = (\overset{x_2}{12}, \overset{y_2}{8})$

Slope ⇒ $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{8-6}{12-4} = \dfrac{2}{8} = \dfrac{1}{4}$

$$\lim_{x_1 \to x_2} \frac{f(x_2) - f(x_1)}{x_2 - x_1} \rightarrow h = x_2 - x_1$$

$$x^2 = h + x_1$$

$$\lim_{h \to 0} \frac{f(h + x_1) - f(x_1)}{h}$$

2.8 The Derivative as a Function

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

If $f(x) = x^3 - x$, find a formula for $f'(x)$.

$$f'(x) = \lim_{h\to 0} \frac{[(x+h)^3 - (x+h)] - [x^3 - x]}{h}$$

$$\lim_{h\to 0} \frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 - \cancel{x} - h - \cancel{x^3} + \cancel{x}}{h} = \lim_{h\to 0} \frac{3x^2h + 3xh^2 + h^3 - h}{h}$$

$$\lim_{h\to 0} \frac{\cancel{h}(3x^2 + 3xh + h^2 - 1)}{\cancel{h}} = \lim_{h\to 0} 3x^2 + 3\cancel{xh}^{0} + \cancel{h^2}^{0} - 1 = 3x^2 - 1$$

$$f(x) = x^3 - x^1$$

$$f'(x) = 3.(x^{3-1}) - 1.(x^{1-1})$$

$$x^0 = 1$$

$$= 3x^2 - 1$$

If $f(x) = x^3 - x$, find a formula for $f'(x)$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{[(x+h)^3 - (x+h)] - (x^3 - x)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 \cancel{-x} - h \cancel{-x^3} \cancel{+x}}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{3x^2h + 3xh^2 + h^3 - h}{h} \Rightarrow \lim_{h \to 0} \frac{\cancel{h}(3x^2 + 3xh + h^2 - 1)}{\cancel{h}}$$

$$\lim_{h \to 0} 3x^2 + 3x\overset{0}{\cancel{h}} + \overset{0}{\cancel{h}}{}^2 - 1 \Rightarrow f'(x) = 3x^2 - 1$$

If $f(x) = \sqrt{x}$, find the derivative of f. State the domain of f'.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$$

$$f'(x) = \lim_{h \to 0} \frac{(x+h) - x}{h\left(\sqrt{x+h} + \sqrt{x}\right)} \Rightarrow \lim_{h \to 0} \frac{\cancel{h}\ 1}{\cancel{h}\left(\sqrt{x+h} + \sqrt{x}\right)}$$

$$f'(x) = \lim_{h \to 0} \frac{1}{\sqrt{x+\cancel{h}} + \sqrt{x}} \Rightarrow f'(x) = \frac{1}{\sqrt{x} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$$

$f(x) = \sqrt{x} \Rightarrow$ Domain $[0, \infty)$, $f'(x) = \frac{1}{2\sqrt{x}} \Rightarrow$ Domain $(0, \infty)$

Find f' if $f(x) = \dfrac{1 - x}{2 + x}$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\boxed{\dfrac{1 - (x+h)}{2 + (x+h)} - \dfrac{1-x}{2+x}}}{h} \rightarrow$$ توحيد المقام

$$f'(x) = \lim_{h \to 0} \frac{\boxed{\dfrac{(2+x)(1-x+h) - (2+x+h)(1-x)}{(2+x)(2+x+h)}}}{h}$$

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{ad}{bc}$$

$$f'(x) = \lim_{h \to 0} \frac{(2+x)(1-x+h) - (2+x+h)(1-x)}{h(2+x)(2+x+h)}$$

$$f'(x) = \lim_{h \to 0} \frac{(1-x-h)(2+x) - (1-x)(2+x+h)}{h(2+x+h)(2+x)}$$

$$f'(x) = \lim_{h \to 0} \frac{\cancel{2} - \cancel{x} - 2h - \cancel{x^2} - \cancel{xh} \cancel{-2} + \cancel{x} - h + \cancel{x^2} + \cancel{xh}}{h(2+x+h)(2+x)}$$

$$f'(x) = \lim_{h \to 0} \frac{-3\cancel{h}}{\cancel{h}(2+x+h)(2+x)} = \lim_{h \to 0} \frac{-3}{(2+x+\overset{0}{\cancel{h}})(2+x)}$$

$$f'(x) = \frac{-3}{(2+x)(2+x)} = \frac{-3}{(2+x)^2}$$

3.1 Derivatives of Polynomials and Exponential Functions

The Power Rule (General Version) If n is any real number, then

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

x^2

$2 \cdot (x^{2-1}) = 2x$

If $f(x) = x^6$, then $f'(x) = 6x^5$.

If $y = x^{1000}$, then $y' = 1000x^{999}$.

If $y = t^4$, then $\frac{dy}{dt} = 4t^3$.

Derivative of a Constant Function

$$\frac{d}{dx}(c) = 0$$

$f(x) = 3$, find $f'(x) = 0$

The Sum and Difference Rules If f and g are both differentiable, then

$$\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

$$\frac{d}{dx}[f(x) - g(x)] = \frac{d}{dx}f(x) - \frac{d}{dx}g(x)$$

$f'(x)$, y'

$\frac{d}{dx}$

$$\frac{d}{dx}(x^8 + 12x^5 - 4x^4 + 10x^3 - 6x + 5) \Rightarrow \frac{d}{dx}x^8 + \frac{d}{dx}12x^5 - \dots$$

$$8x^7 + 12\cdot(5)x^4 - 4(4)x^3 + 10(3)x^2 - 6(1) + 0$$

$$8x^7 + 60x^4 - 16x^3 + 30x^2 - 6$$

Derivative of the Natural Exponential Function

$$\frac{d}{dx}(e^x) = e^x$$

Derivative of the Exponential Function

$$y = b^{f(x)} \Rightarrow y' = b^{f(x)} \ln(b)\, f'(x)$$

Ex ① : $y = 2^x$

$y' = 2^x \ln(2)\ (1)$

$= 2^x \ln(2)$

Ex ② : $y = 7^{(3x + x^2)}$

$y' = 7^{(3x+x^2)} \ln(7)$

$\cdot\ (3(1) + 2x')$

$= 7^{(3x+x^2)} \ln(7)\ (3 + 2x)$

3–34 Differentiate the function.

3. $g(x) = 4x + 7$

5. $f(x) = x^{75} - x + 3$

10. $r(z) = z^{-5} - z^{1/2}$

18. $W(t) = \sqrt{t} - 2e^t$

20. $F(t) = (2t - 3)^2$

41–42 Find equations of the tangent line and normal line to the curve at the given point.

42. $y = x^{3/2}, \quad (1, 1)$