OL Academy

Lesson 1

PHYCS102

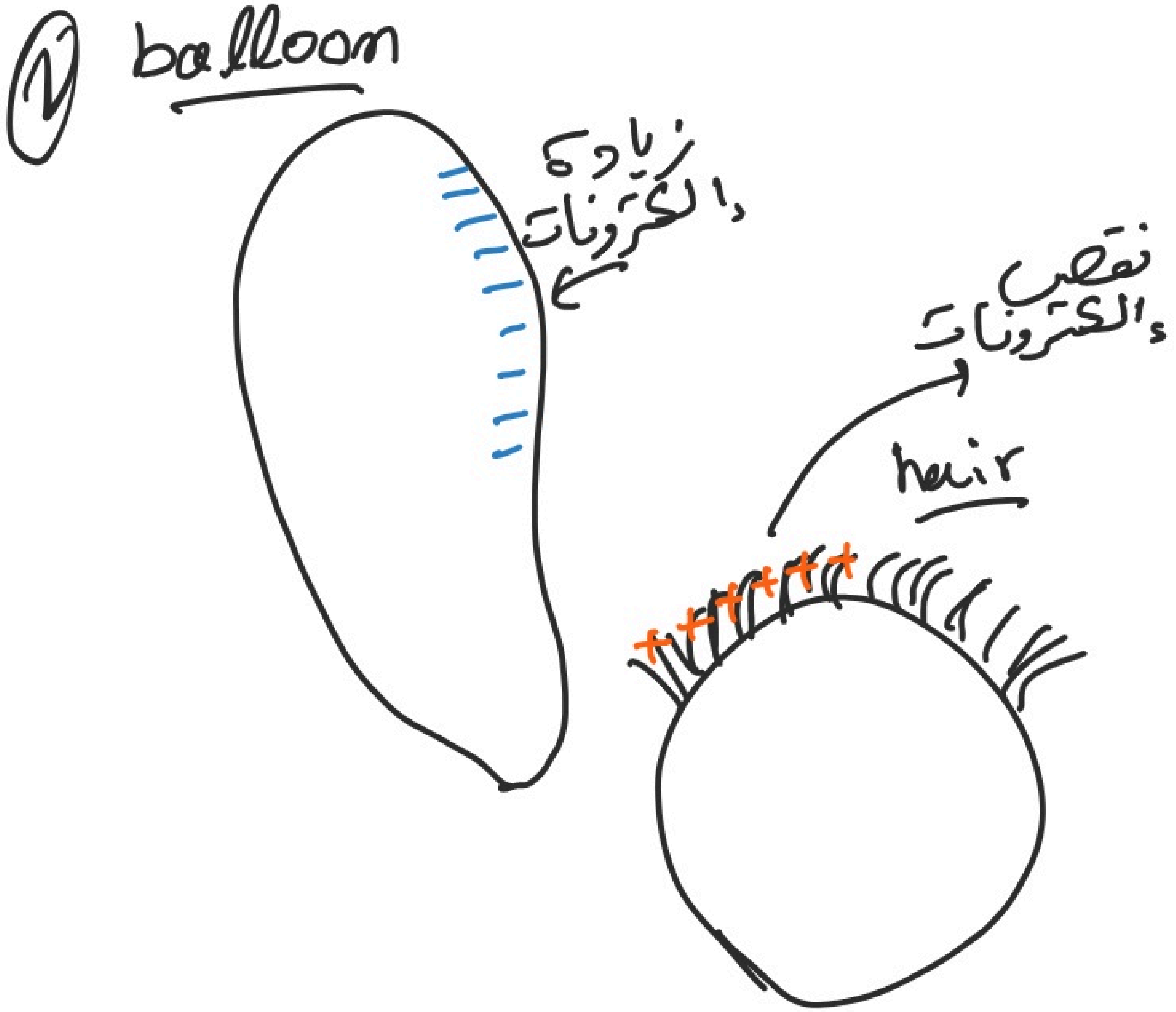
Chapter 22 | Electric Fields

22.3 Coulomb's Law

T. Sayed Ali Madan


atom:

1



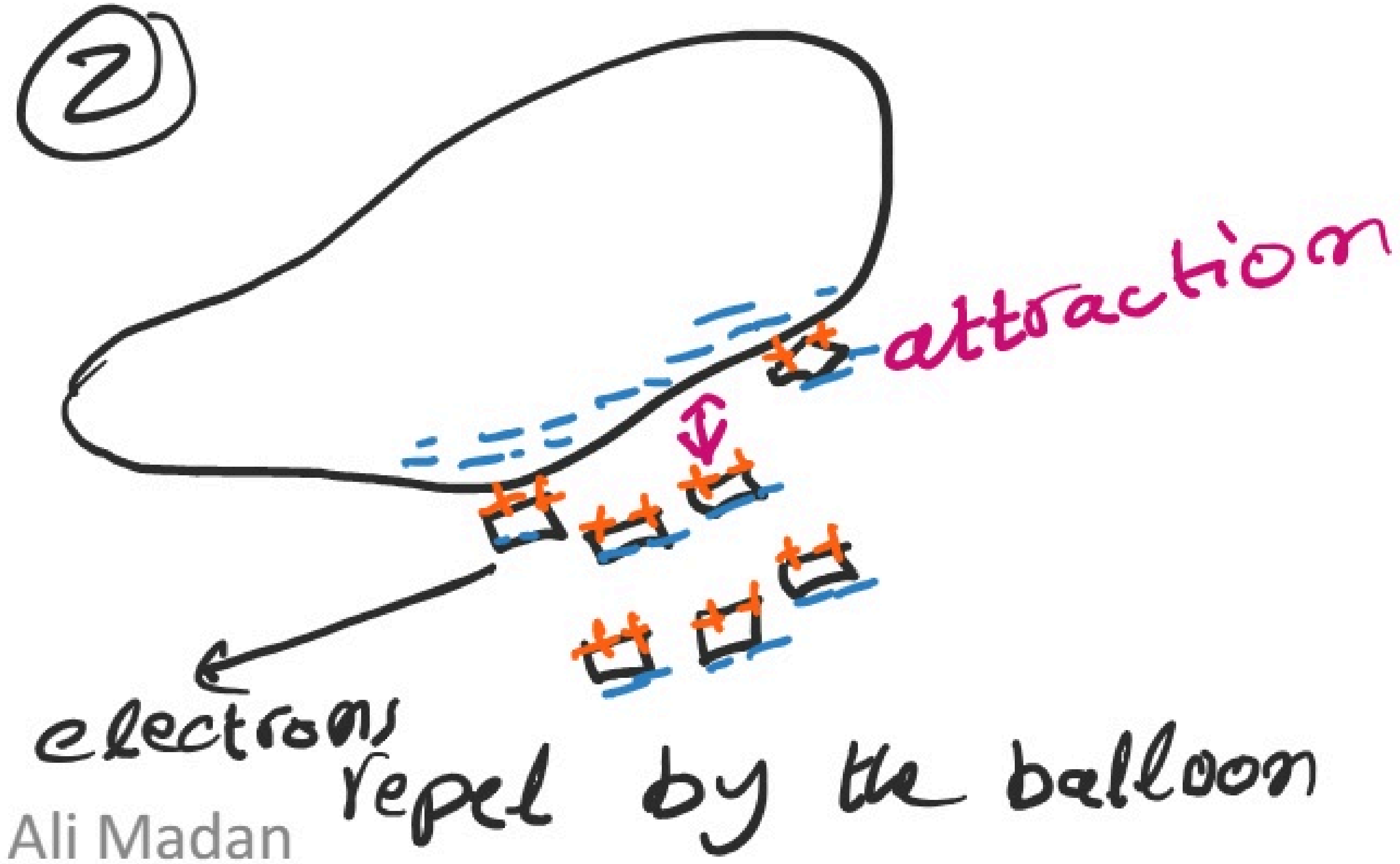
* electrons moved from hair to balloon.

* smallest charge is electron charge

$$e = \underline{1.6 \times 10^{-19}}\ \underline{C}$$

C : Coulomb

2



Outline

- Revision of vectors
- Coulomb's Law
- Examples

Outline

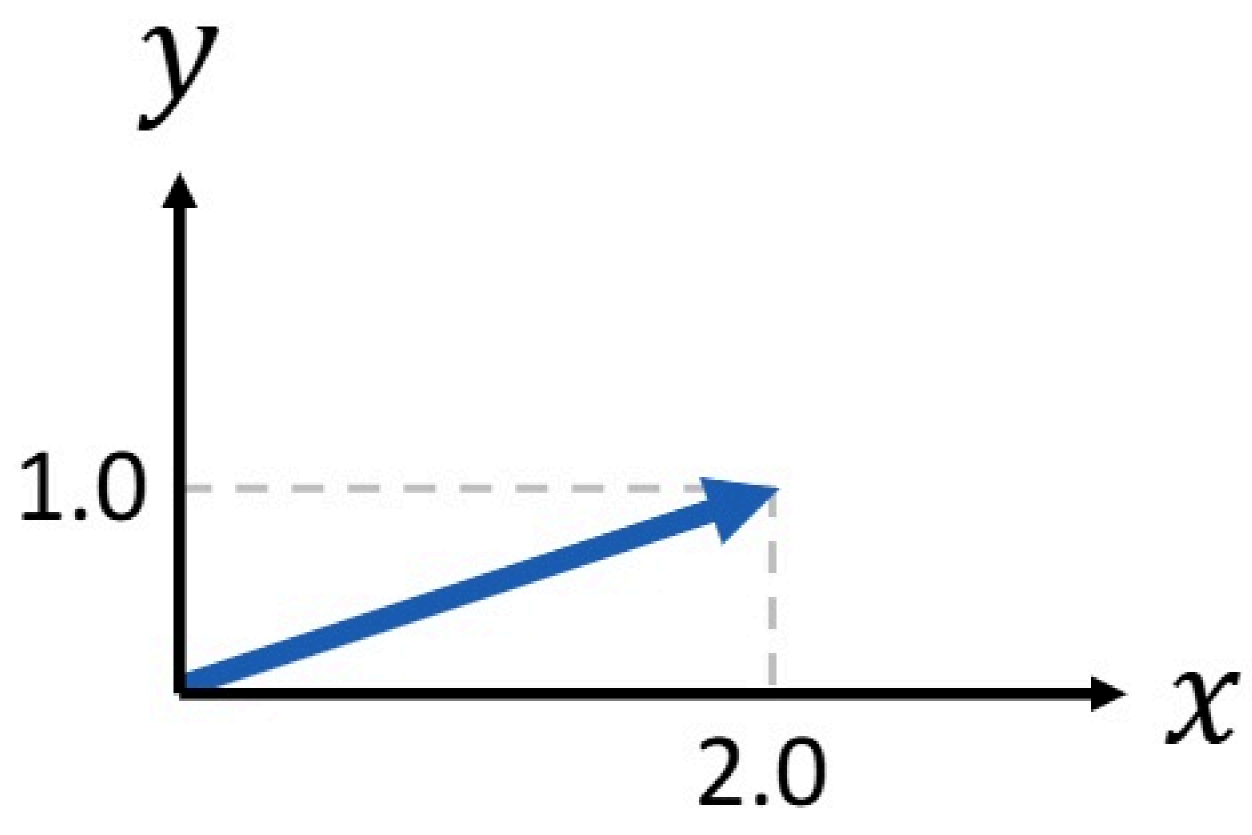
- **Revision of vectors**
- Coulomb's Law
- Examples

Revision: Vectors

$\vec{F}$ — $\underline{10N}$ down ↓

$\vec{d}$ — $\underline{2m}$ right

A **Vector** quantity has a magnitude and direction.

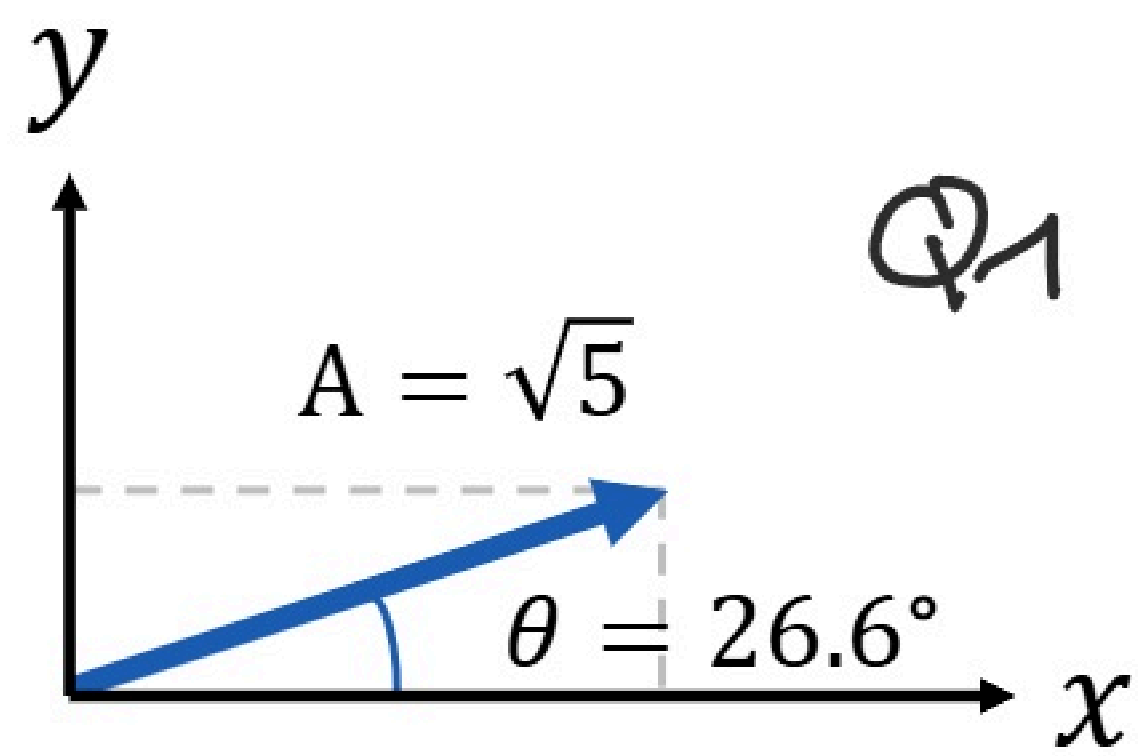


magnitude

$$A = \sqrt{A_x^2 + A_y^2}$$

direction

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$



Q1

Note: θ is taken counterclockwise from the +x direction

$$A_x = \sqrt{5}\cos 26.6^\circ = 2$$

y-component

$$A_y = A\sin\theta$$

x-component

$$A_x = A\cos\theta$$

$$A_x = 2.0$$

$$A_y = 1.0$$

$$A_y = \sqrt{5}\sin 26.6^\circ = 1$$

$$\theta = \tan^{-1}\left(\frac{1.0}{2.0}\right) \underbrace{+ 0}_{Q_1} = 26.6^\circ$$

$$A = \sqrt{2.0^2 + 1.0^2} = 2.2 = \sqrt{5}$$

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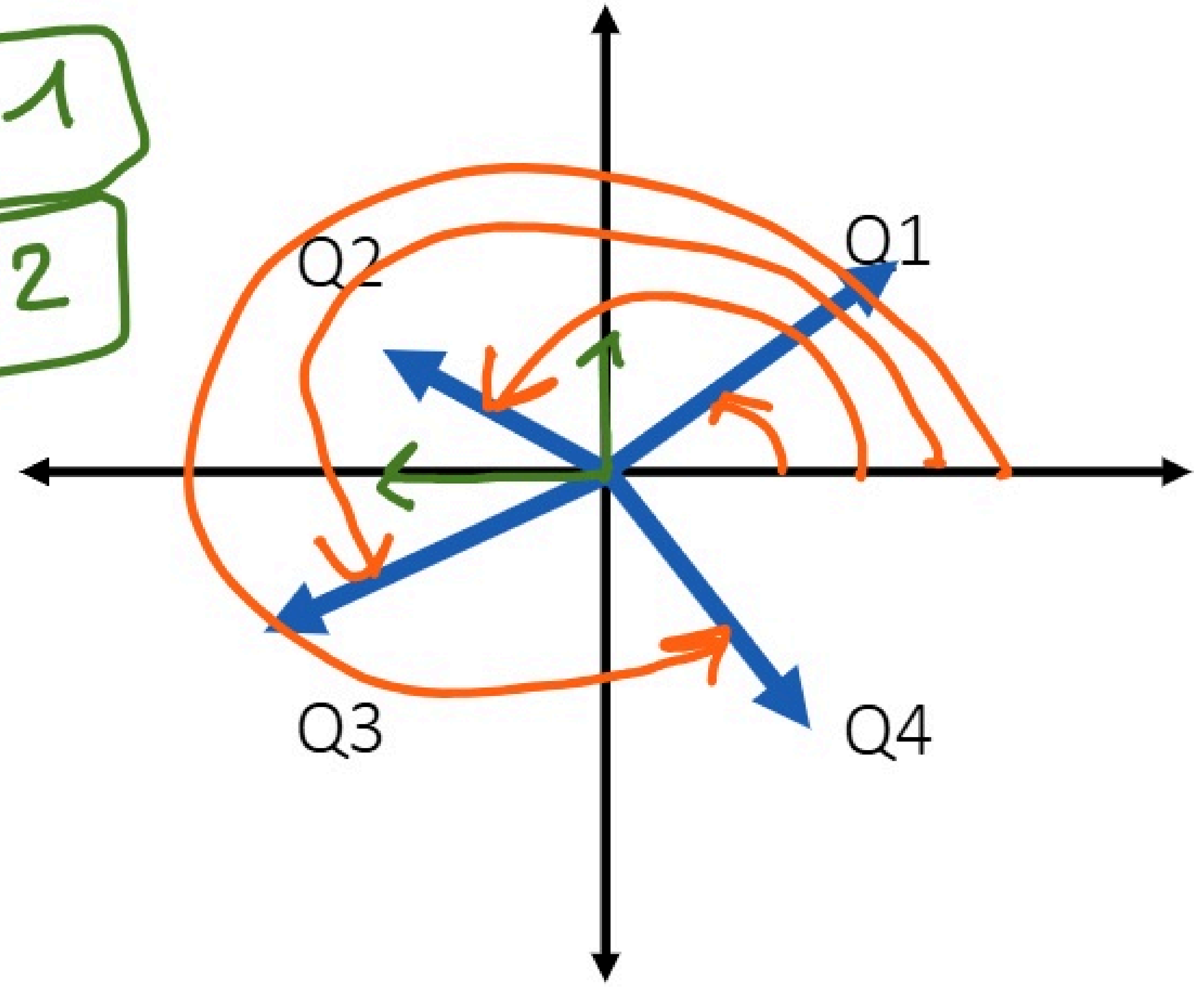
with signs!

How to calculate $\boldsymbol{\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)}$ using the calculator:

$\boxed{A_y = +1}$

$\boxed{A_x = -2}$



Q1: $\theta = (calculator\ result) + 0$

Q2: $\theta = (calculator\ result) + 180^\circ$

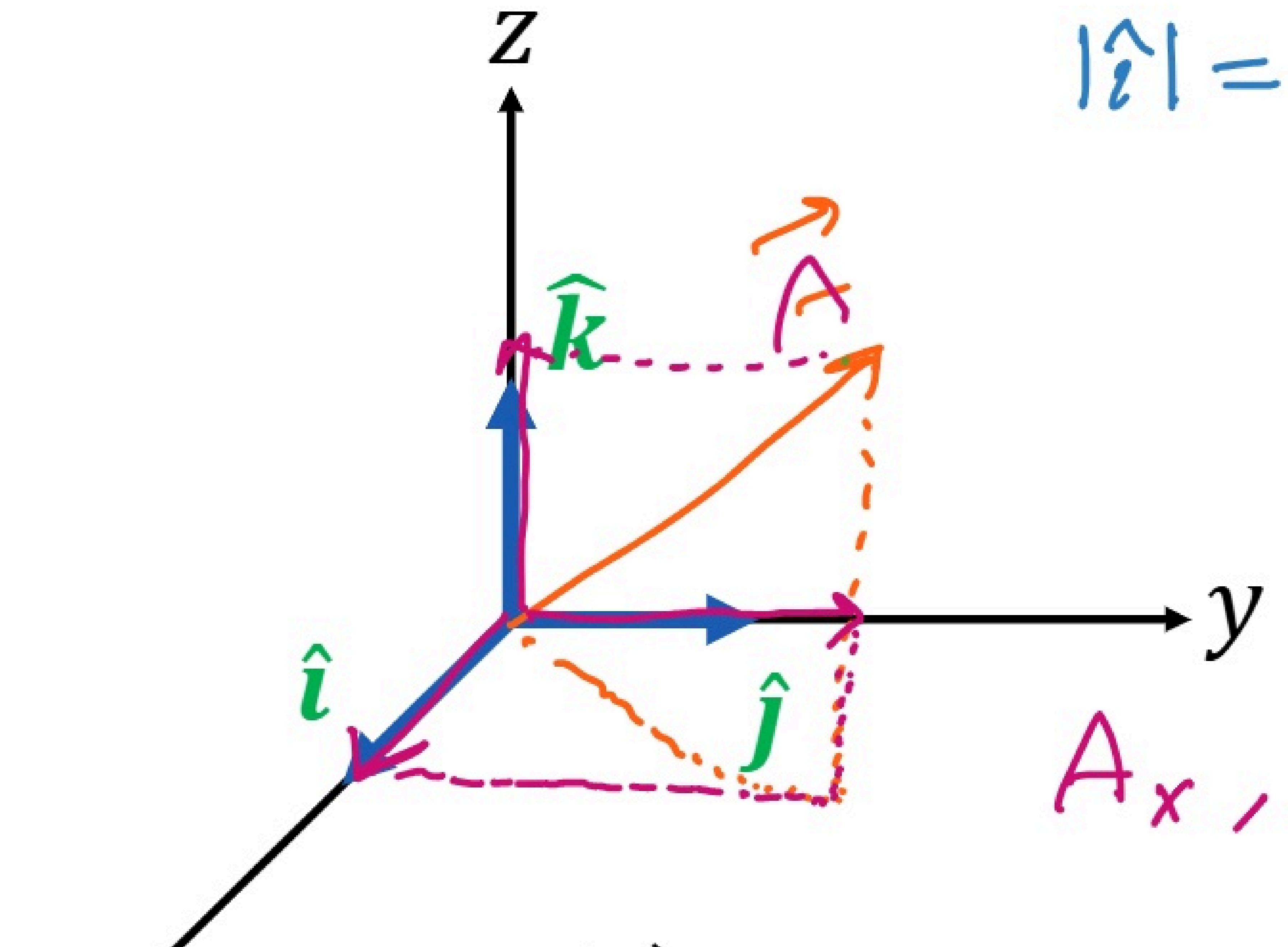
Q3: $\theta = (calculator\ result) + 180^\circ$

Q4: $\theta = 360^\circ + (calculator\ result)$

→ You'll get θ to be taken counterclockwise from the +x direction

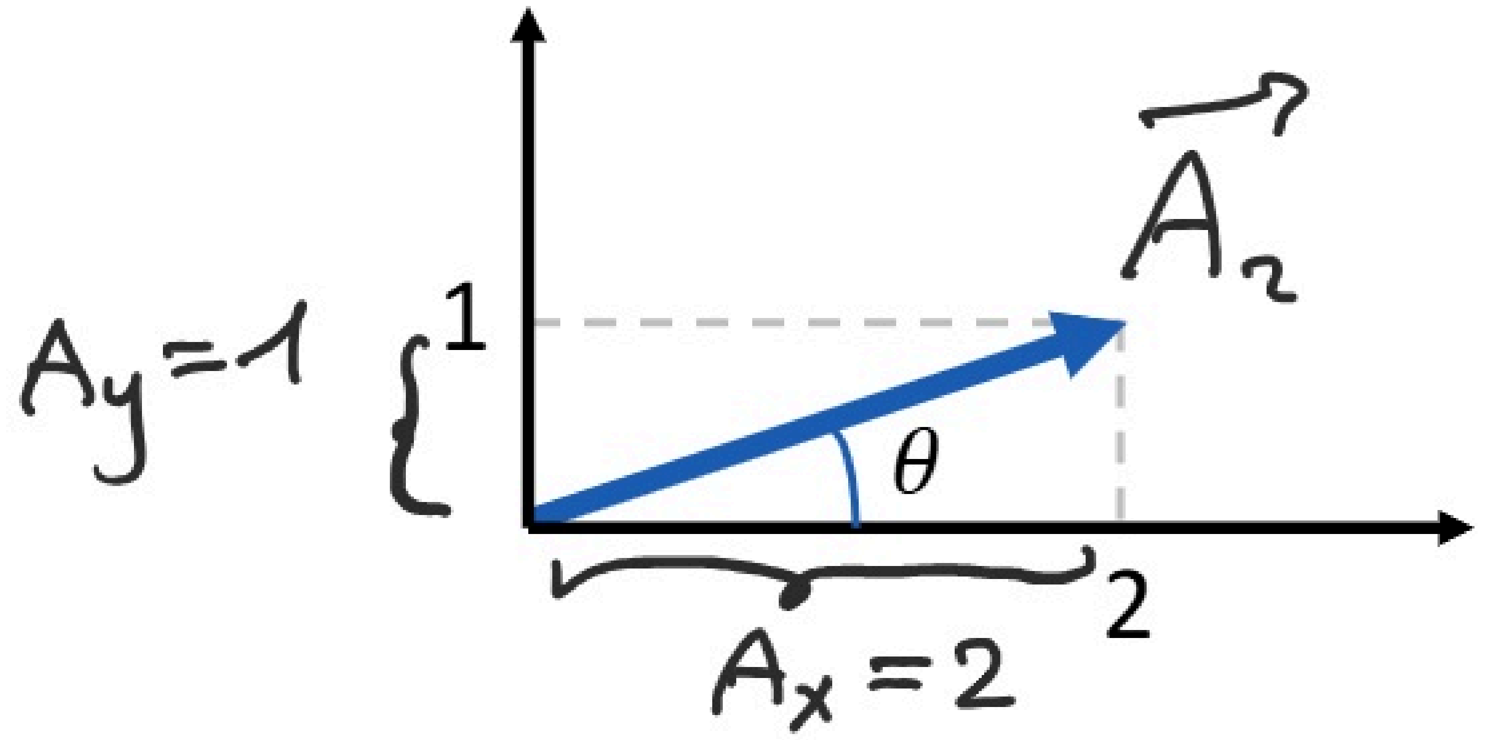
Unit vector notation:

$|\hat{i}| = |\hat{j}| = |\hat{k}| = 1$



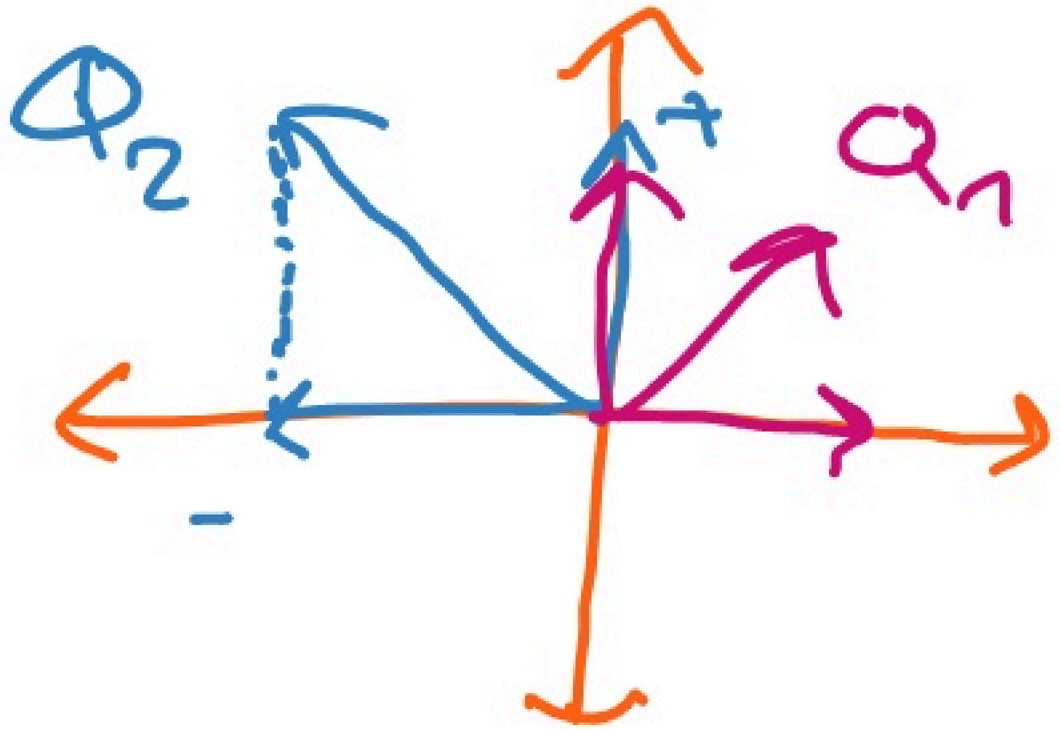
A_x, A_y, A_z

$$\vec{A}_1 = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$



$$\vec{A} = A_x\hat{i} + A_y\hat{j}$$
$$= 2\hat{i} + \hat{j}$$

Resultant of vectors:



Find the resultant (net/sum) of the following two vectors:

$$\overrightarrow{A_1} = -4\hat{\imath} + 3\hat{\jmath}, \qquad \overrightarrow{A_2} = -\hat{\imath} + \hat{\jmath}$$

$$\vec{A}_{resultant} = \vec{A}_1 + \vec{A}_2 = (-4\hat{\imath} + 3\hat{\jmath}) + (-\hat{\imath} + \hat{\jmath})$$
$$= -5\hat{\imath} + 4\hat{\jmath}$$

Magnitude and direction of the resultant:

$$A_{resultant} = \sqrt{A_x^2 + A_y^2} = \sqrt{(-5)^2 + (4)^2} = 6.4, \quad \theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$
$$= \tan^{-1}\left(\frac{4}{-5}\right) = -38.7 + \underbrace{180}_{\theta_2}$$
$$= 141.3$$

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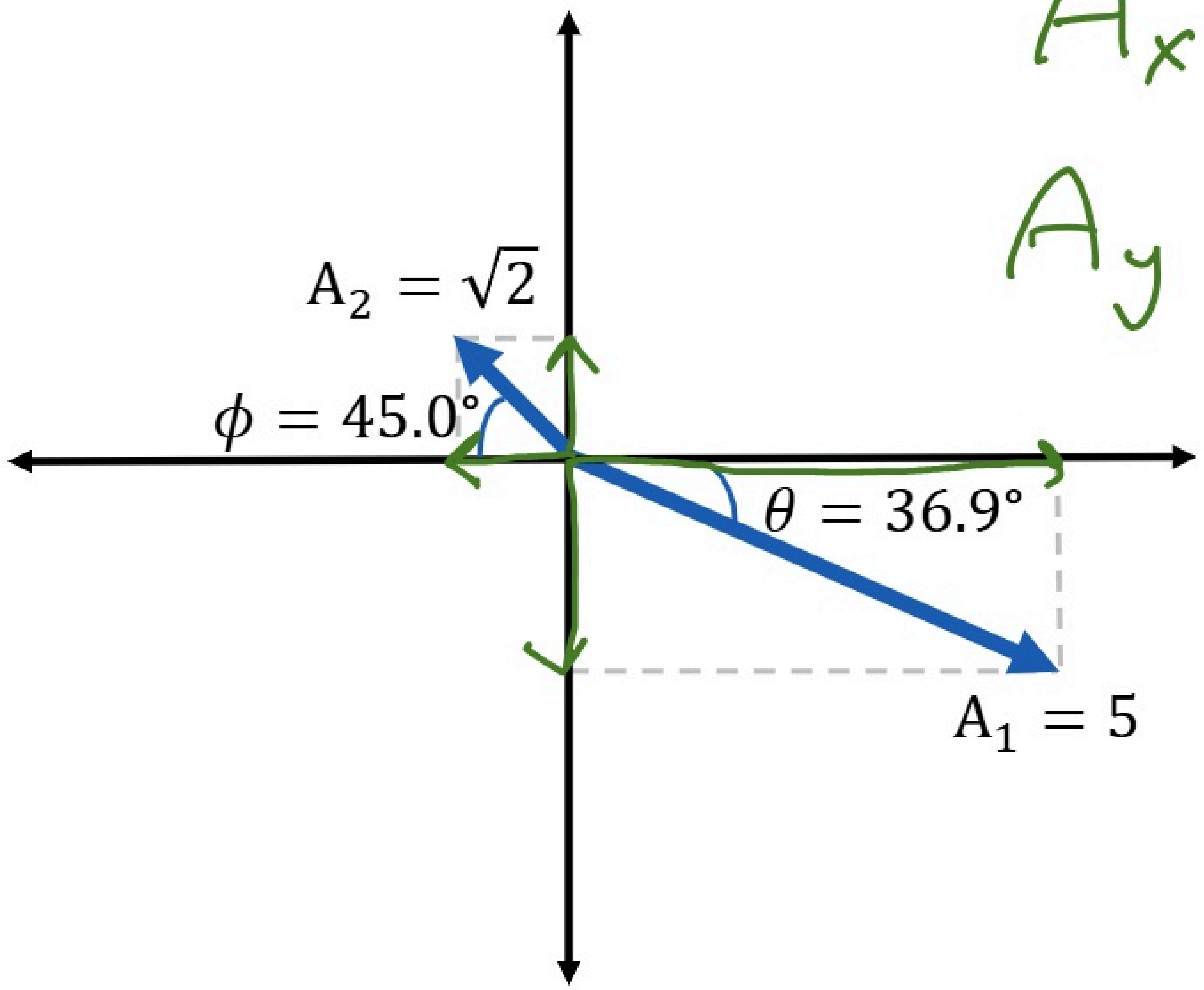
Revision: Vectors

Find the resultant of the following two vectors:

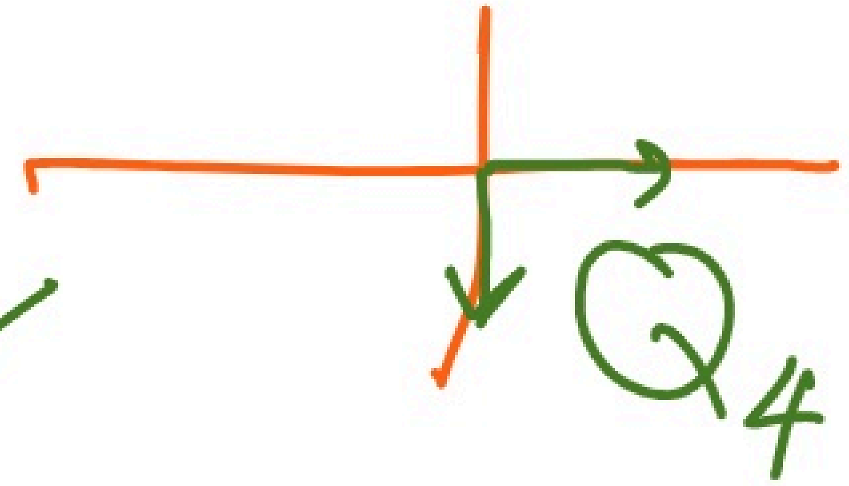
$$\vec{A}_{resultant} = A_x\hat{i} + A_y\hat{j}$$



$$A_x = A_{1,x} + A_{2,x} = +5\cos 36.9° - \sqrt{2}\cos 45° = 3$$

$$A_y = A_{1,y} + A_{2,y} = -5\sin 36.9 + \sqrt{2}\sin 45° = -2$$

$$\vec{A}_{resultant} = 3\hat{i} - 2\hat{j}$$



If we need magnitude and direction:

$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(3)^2 + (-2)^2} = \sqrt{13}$$

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) = \tan^{-1}\left(\frac{-2}{3}\right) = -33.7 + 360 = 326.3°$$

Revision: Vectors

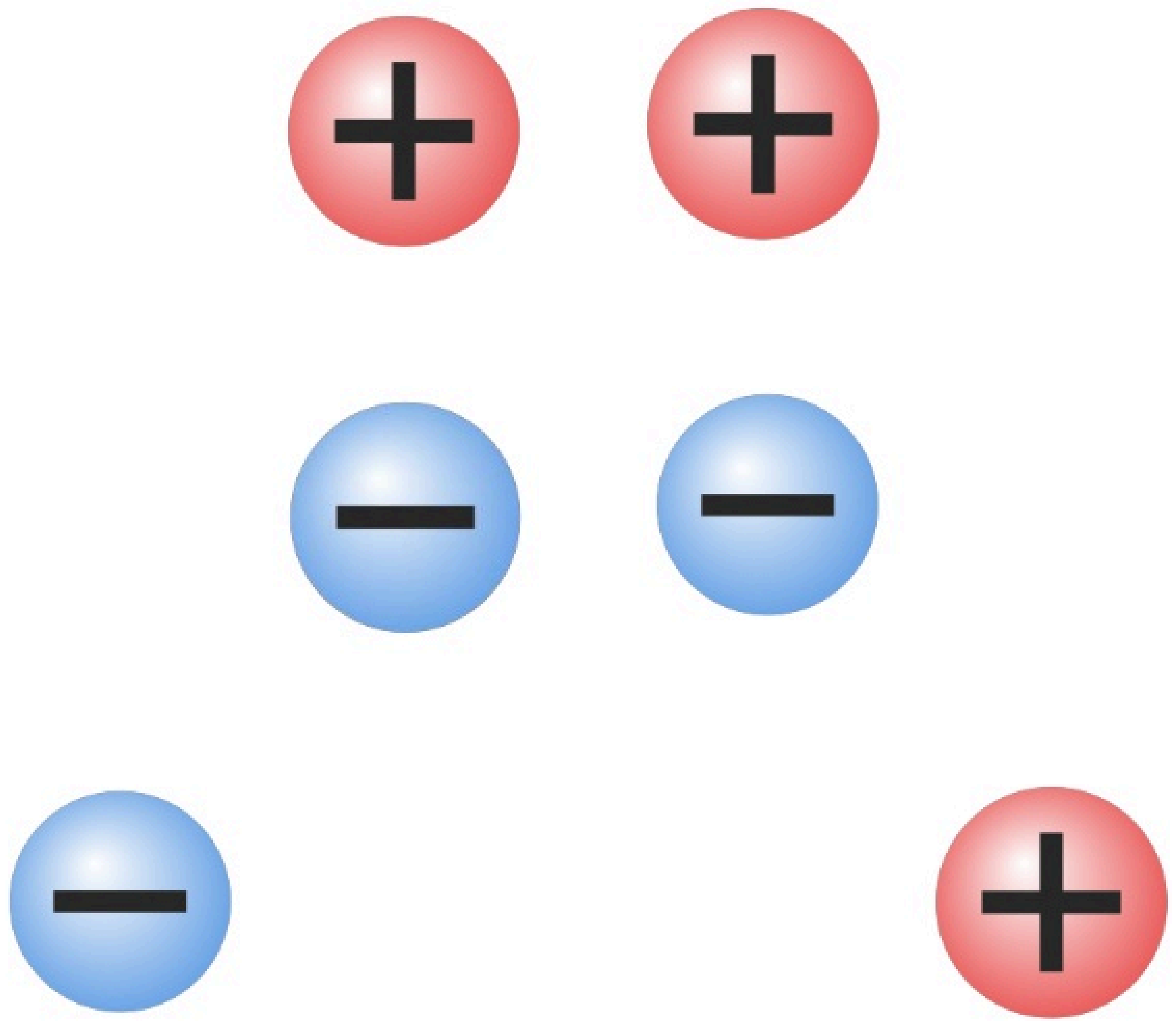
Resultant of n vectors:

$$\overrightarrow{\boldsymbol{A}} = \overrightarrow{\boldsymbol{A_1}} + \overrightarrow{\boldsymbol{A_2}} + \cdots + \overrightarrow{\boldsymbol{A_n}}$$

Outline

- Revision of vectors
- **Coulomb's Law**
- Examples

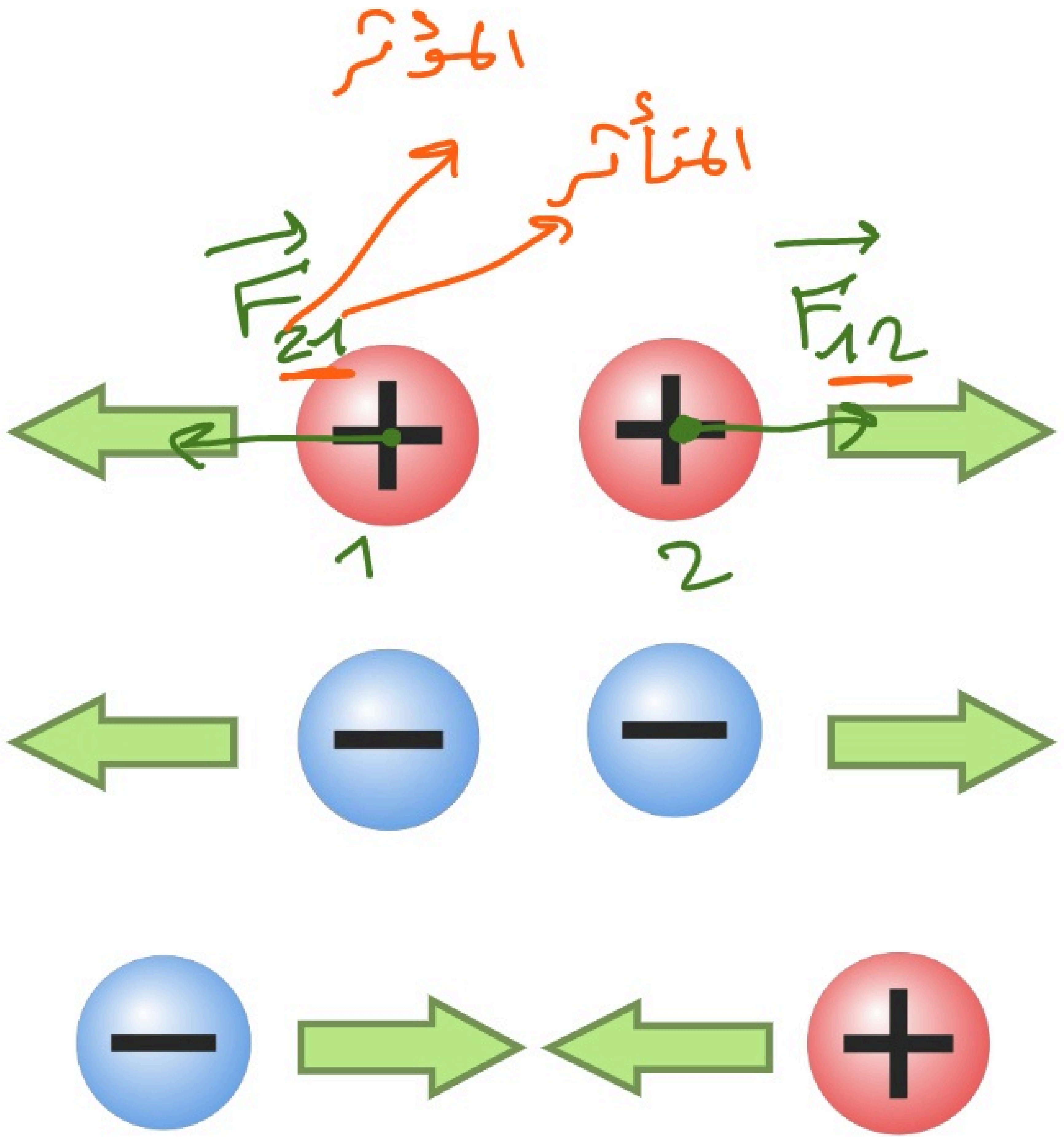
Electric force between two point charges:



Section 22.3: Coulomb's Law

Direction of electric force:

- Similar repels
- Different attracts



Electric forces follow Newton's Third Law:

$$\vec{F}_{12} = -\vec{F}_{21}$$

Magnitude of electric force between two charges:

$\boldsymbol{F_e}$: electric force

$\boldsymbol{q_1}$: charge 1

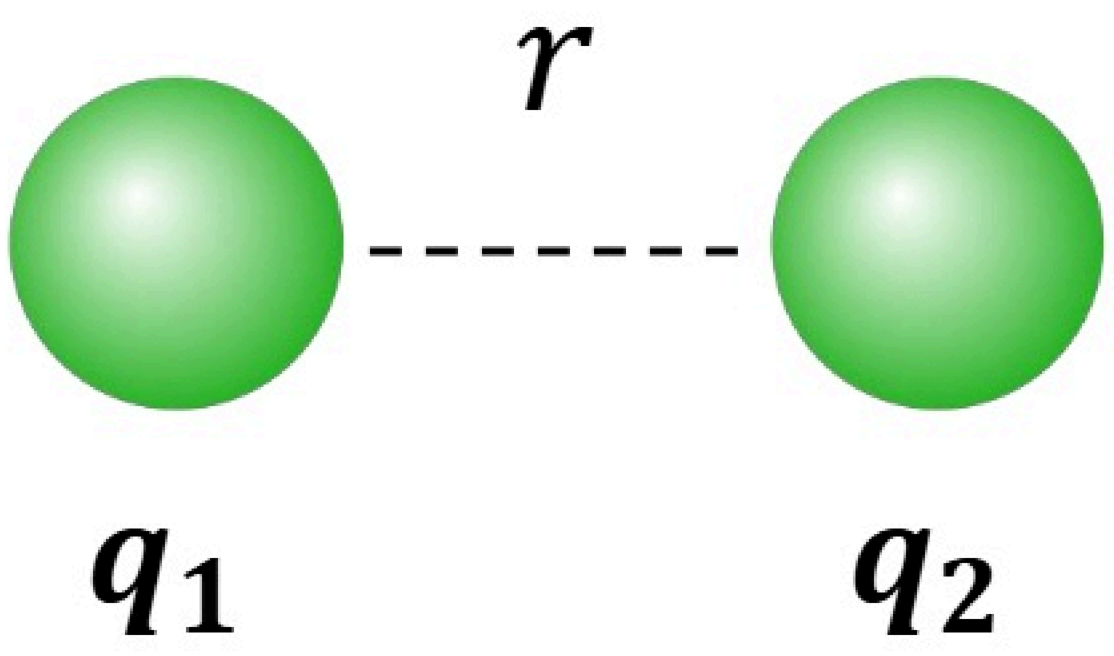
$\boldsymbol{q_2}$: charge 2

$\boldsymbol{r}$: distance between the two charges

$$F_e \propto |q_1|$$

$$F_e \propto |q_2|$$

$$F_e \propto \frac{1}{r^2}$$



Coulomb's Law:

$$F_e = k\frac{|q_1||q_2|}{r^2}$$

for point charges

$k = 8.987 \times 10^9\ N \cdot m^2/C^2$

$$k = \frac{1}{4\pi\epsilon_0}$$

$\epsilon_0 = 8.85 \times 10^{-12}\ \cancel{C^2/N \cdot m^2}$ $\frac{C^2}{N \cdot m^2}$

F_c: Electric force

q_1: Charge 1

q_2: Charge 2

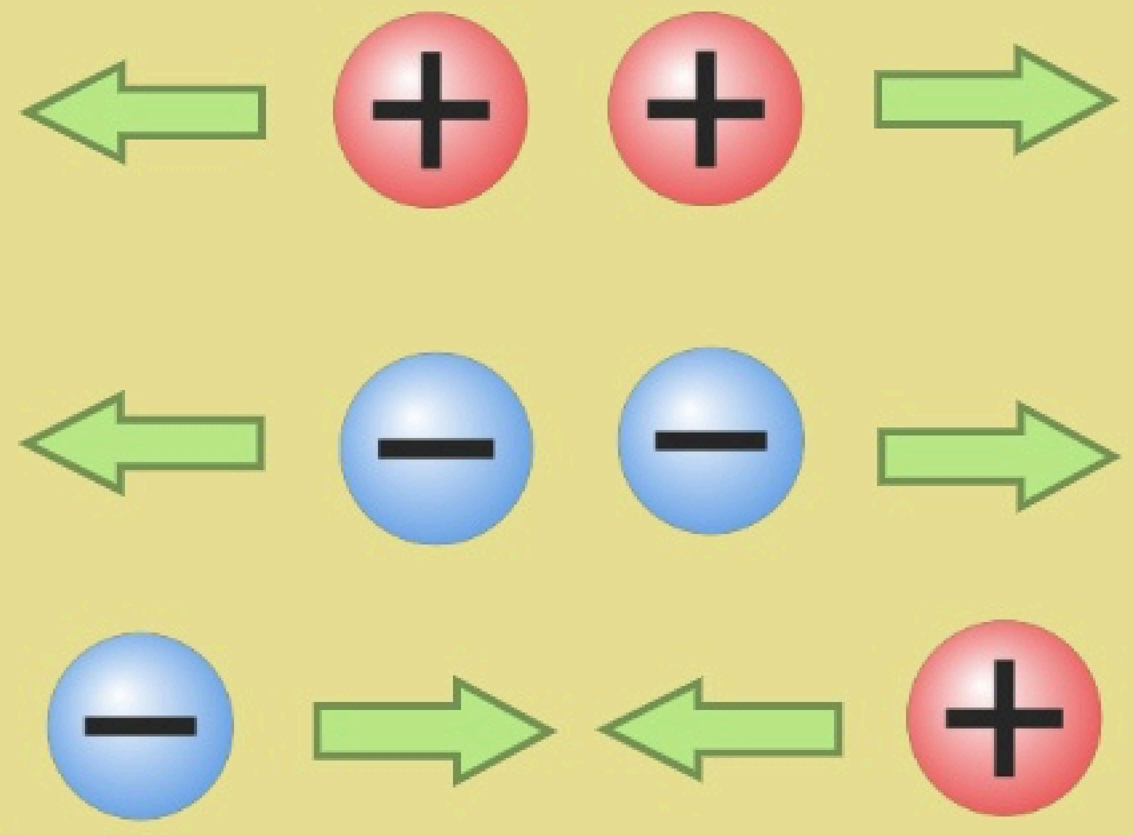
r: Distance between the two charges

k: Coulomb constant

ϵ_0: Permittivity of free space

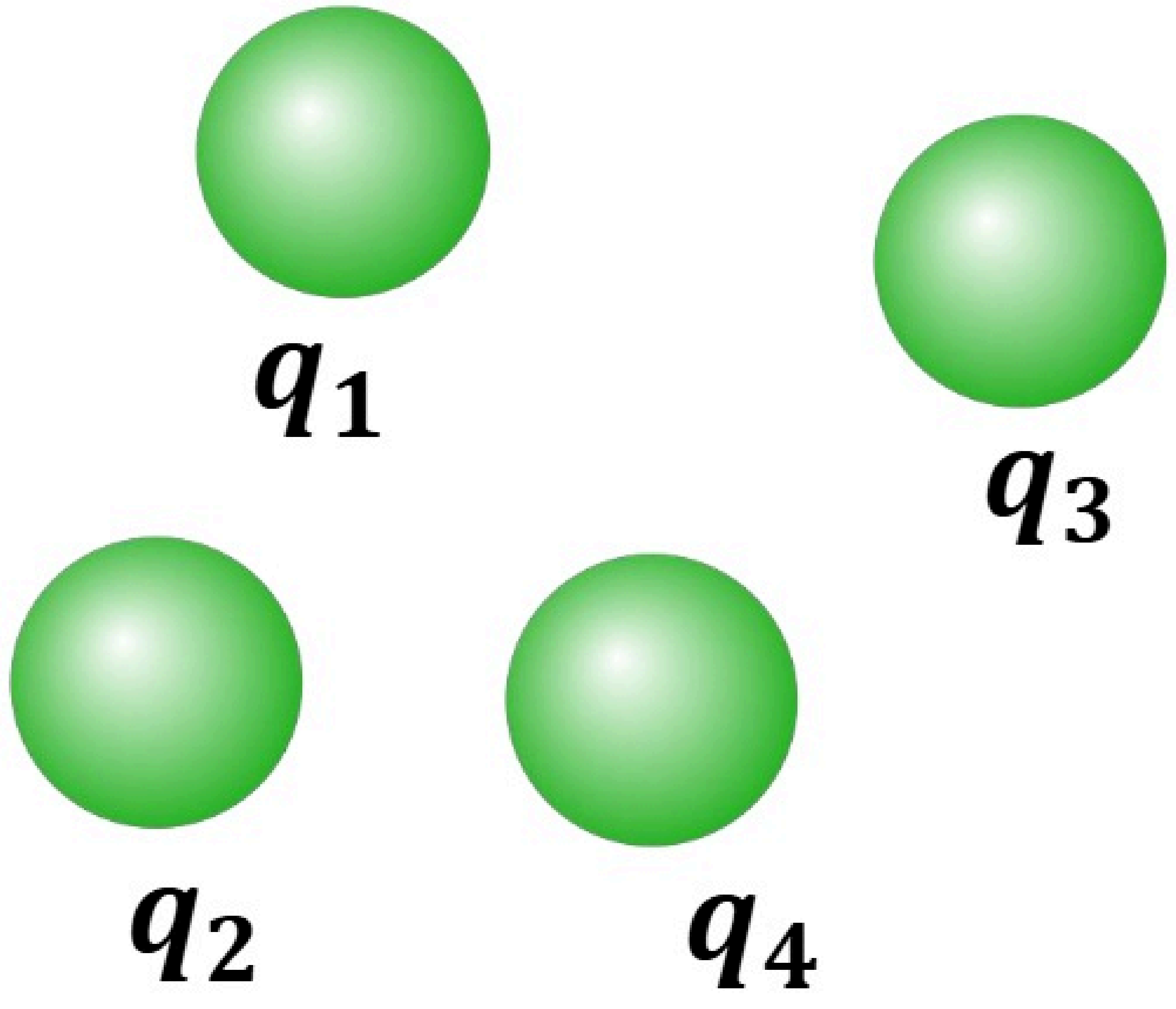
Recall: Rule of thumb:

- Similar repels
- Different attracts



If there are more than two charges, then the electric force between each pair of them is given by Coulomb's Law:

$$F_{23} = k\frac{|q_2||q_3|}{r^2}$$



The net/resultant force exerted on a charge is give by the vector sum of all forces by other charges:

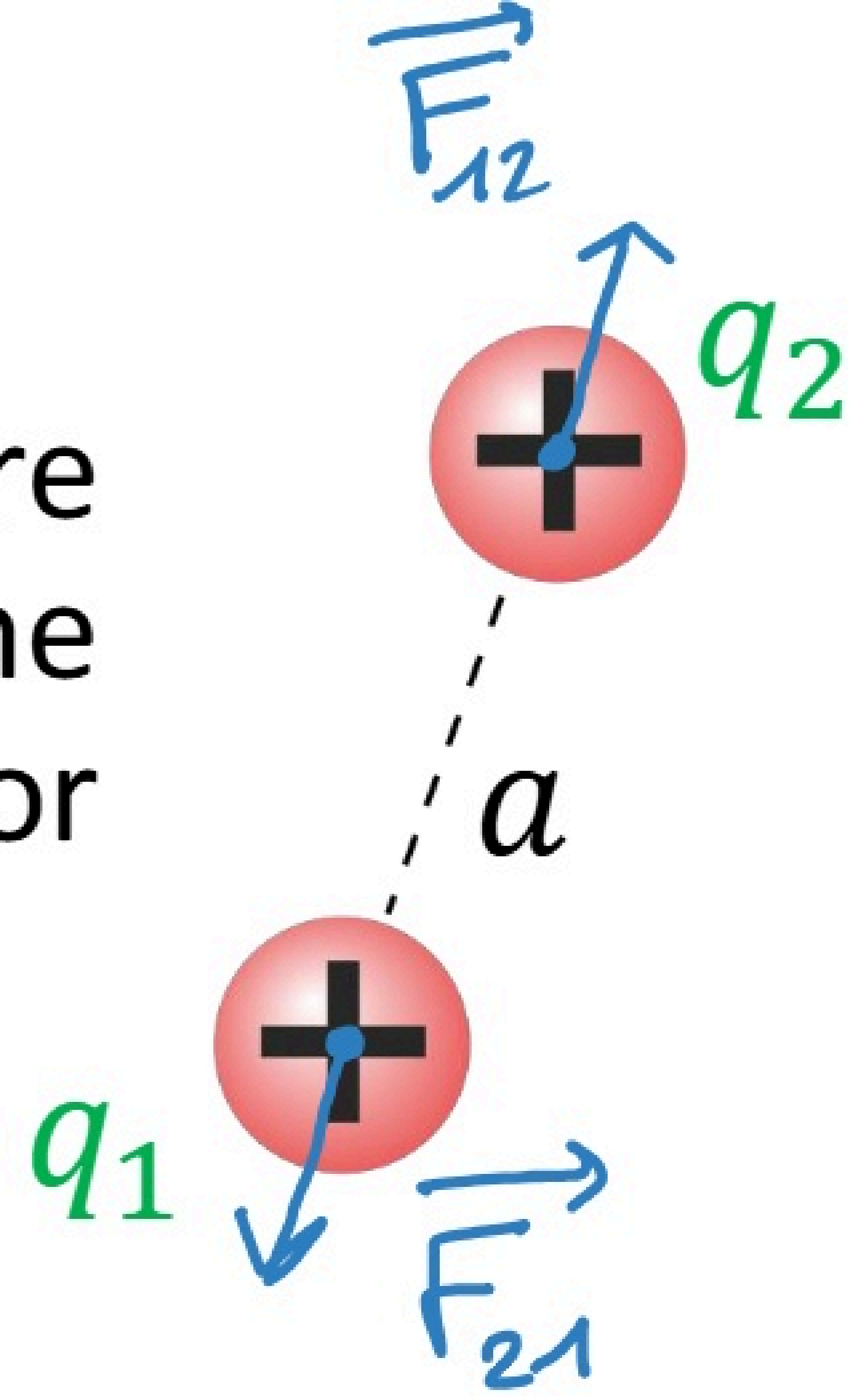
$$\vec{\boldsymbol{F}}_1 = \vec{\boldsymbol{F}}_{21} + \vec{\boldsymbol{F}}_{31} + \vec{\boldsymbol{F}}_{41}$$

Section 22.3: Outline

- Revision of vectors
- Coulomb's Law
- **Examples**

Example 1: Two charges

Consider two **positive** point charges as shown in the figure, where $\boldsymbol{q_1 = 2\mu C}$, $\boldsymbol{q_2 = 3\mu C}$ and $\boldsymbol{a = 0.5m}$. Find the magnitude of the electric force on q_1 by q_2. Is the force between them attractive or repulsive? What is F_{12}?



$\mu = 10^{-6}$

$$F_{21} = \frac{k|q_1||q_2|}{r^2}$$

$$F_{21} = \frac{(8.988 \times 10^{9})(2 \times 10^{-6})(3 \times 10^{-6})}{(0.5)^2} = 0.22N$$

The force is repulsive.

$F_{12} = F_{21} = 0.22N$

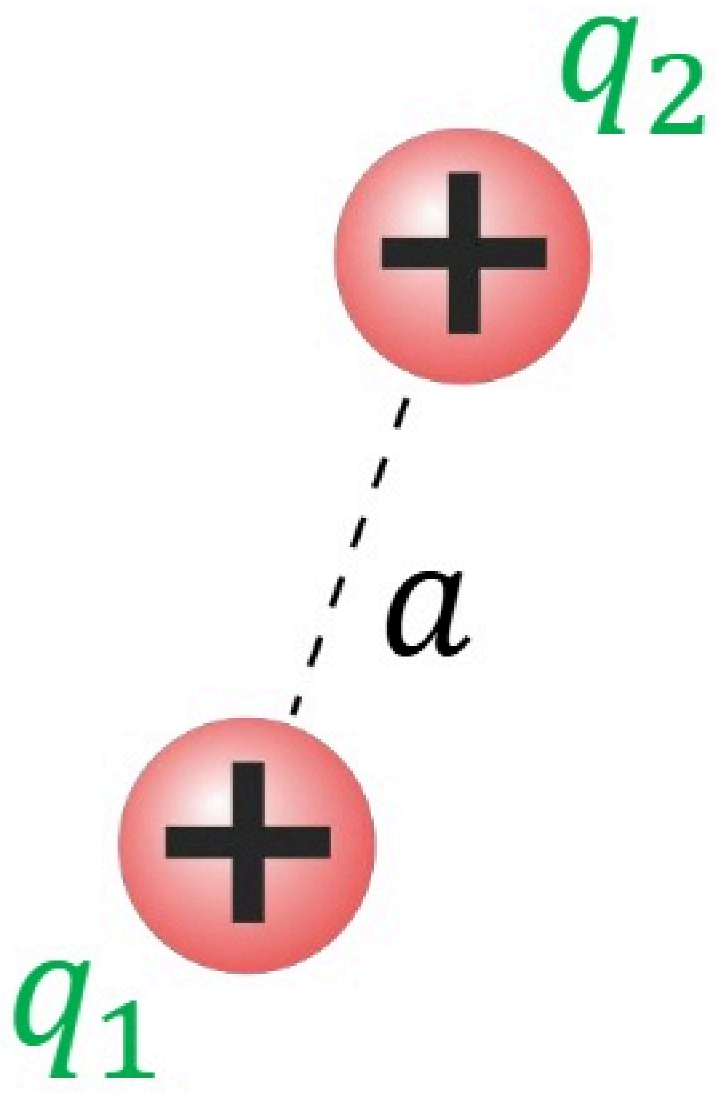
Example 1: Two charges

In the **same** previous configuration, what will happen if:

(a) q_1 increased by double

(b) a is decreased by half

$$F = \frac{k|q_1||q_2|}{r^2}$$



(a) $|q_1| \rightarrow 2|q_1|$ $\quad F \propto |q_1|^1$

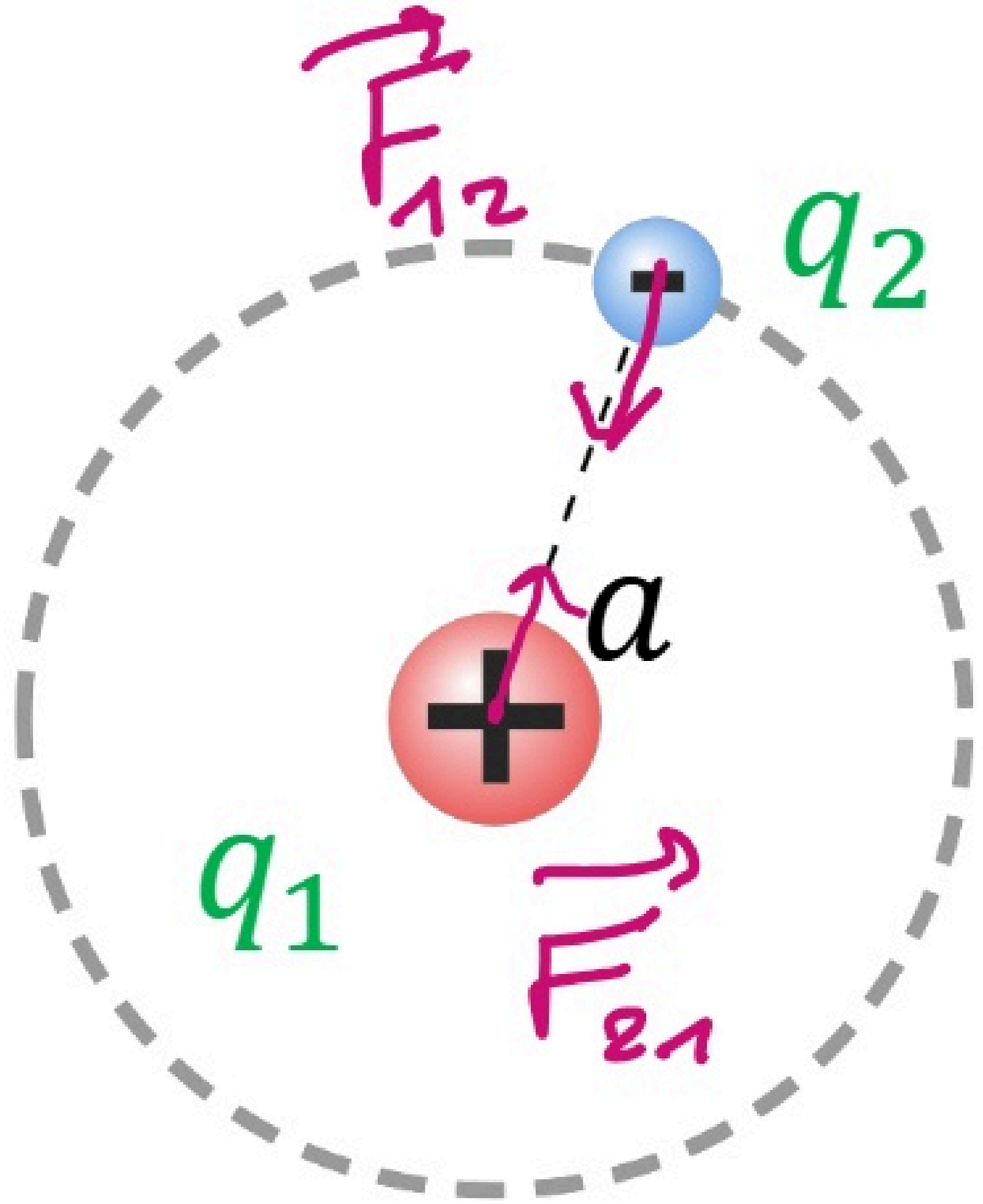
$$F_{new} = 2F_{21} = 2(0.22) = 0.44N$$

(b) $r \rightarrow \frac{1}{2}r$ $\quad F \propto \frac{1}{r^2}$

$$F_{new} = \frac{1}{(\frac{1}{2})^2}F_{21} = 4(0.22) = 0.88N$$

Example 2: The Hydrogen Atom

The **electron** and **proton** of a hydrogen atom are separated by a distance of $\mathbf{a = 5.3 \times 10^{-11}} \boldsymbol{m}$. Find the magnitude of the electric force between the two particles. Is the force between them attractive or repulsive?



Particle	Charge (C)	Mass (kg)
Electron (e)	-1.60×10^{-19}	9.11×10^{-31}
Proton (p)	1.60×10^{-19}	1.67×10^{-27}

$$F = \frac{k|q_1||q_2|}{r^2}$$

$$F = \frac{(8.988 \times 10^9)(1.6 \times 10^{-19})(1.6 \times 10^{-19})}{(5.3 \times 10^{-11})^2}$$

$$= 8.19 \times 10^{-8} N$$

attractive

Example 3: Three charges

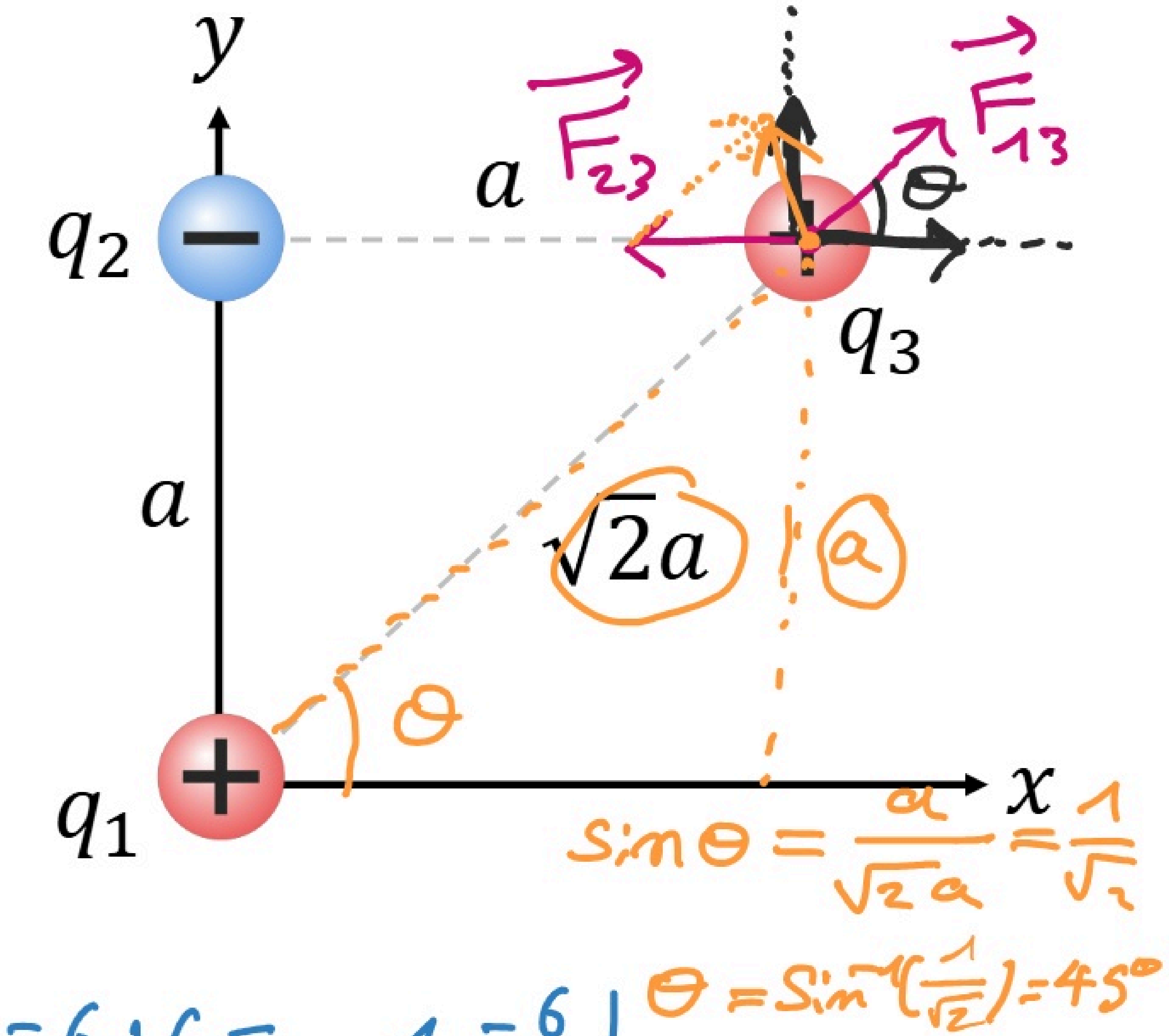
Consider three point charges located at the corners of a right triangle, where $q_1 = q_3 = 5.00\mu C$, $q_2 = -2.00\mu C$ and $a = 0.100\ m$. Find the resultant force exerted on q_3.

magnitude

direction

$\mu = 10^{-6}$

$$\vec{F}_3 = \vec{F}_{13} + \vec{F}_{23}$$



① magnitudes

$$F_{13} = \frac{k|q_1||q_3|}{r^2} = \frac{(8.988\times10^9)(5\times10^{-6})(5\times10^{-6})}{(\sqrt{2}\times0.1)^2} = 11.2\,N$$

$$F_{23} = \frac{k|q_2||q_3|}{r^2} = \frac{(8.988\times10^9)(2\times10^{-6})(5\times10^{-6})}{(0.1)^2} = 8.99\,N$$

② components

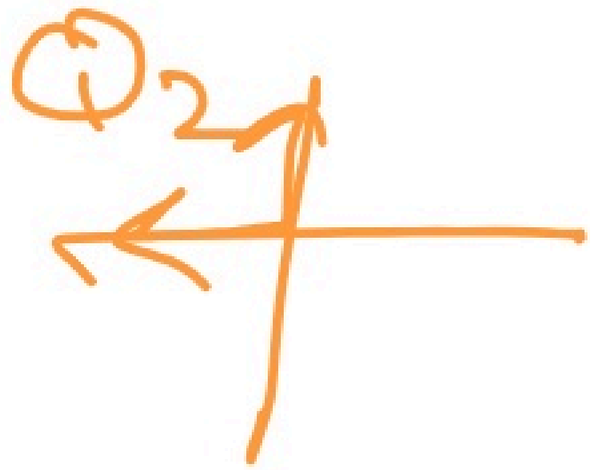
$$F_{13,x} = F_{13}\cos\theta = \underline{11.2\cos 45^\circ}$$

$$F_{13,y} = F_{13}\sin\theta = \underline{11.2\sin 45^\circ}$$

$$F_{3,x} = F_{13,x} + F_{23,x} = 11.2\cos 45^\circ - 8.99 = -1.04\,N$$

$$F_{3,y} = F_{13,y} + F_{23,y} = 11.2\sin 45^\circ + 0 = 7.94\,N$$

$$\vec{F}_3 = (\underline{-1.04}\,\hat{i} + \underline{7.94}\,\hat{j})\,N$$



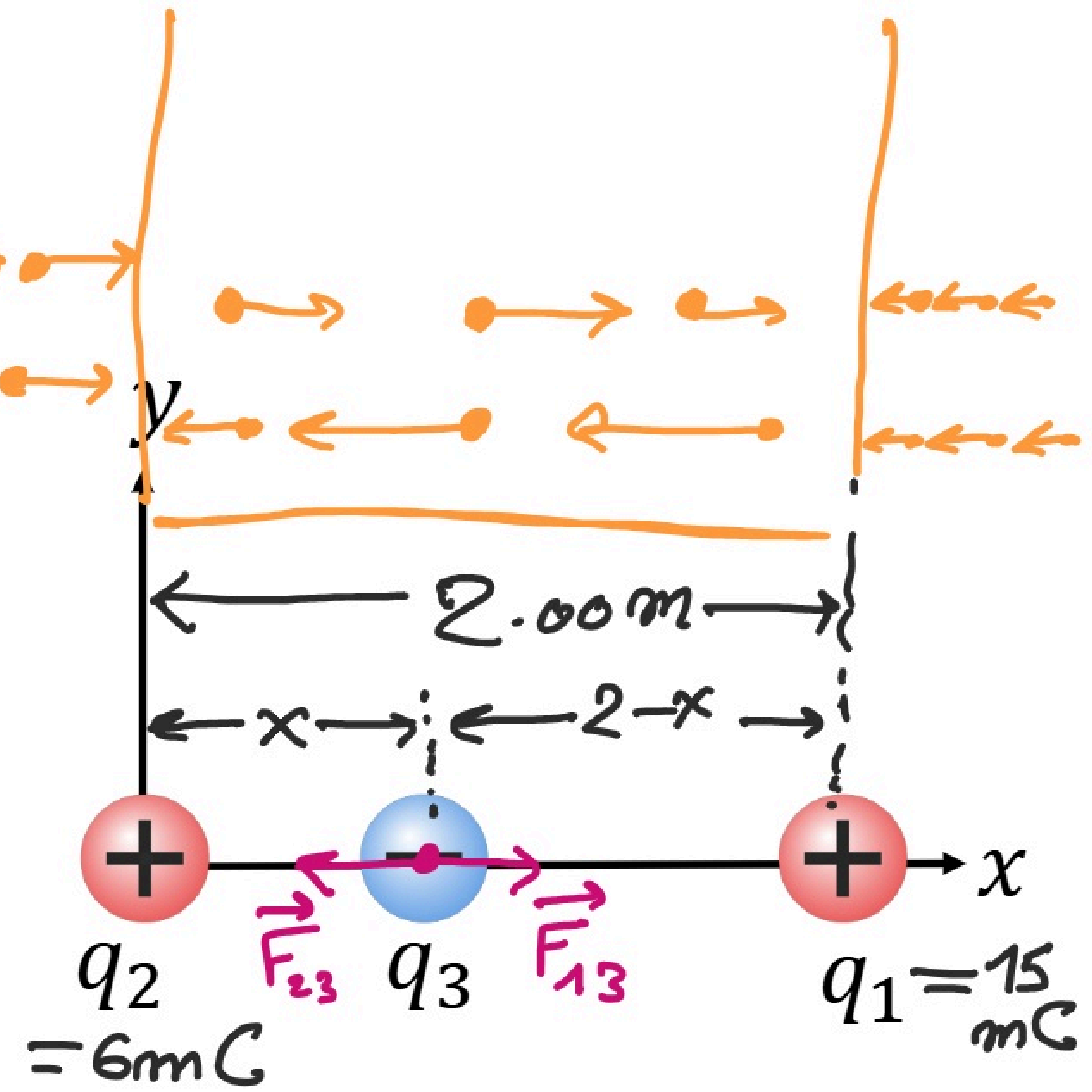
If we need magnitud & direction:

$$F_3 = \sqrt{F_x^2 + F_y^2} = \sqrt{(-1.04)^2 + (7.94)^2} = 8N$$

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = -82.5^\circ + 180^\circ = 97.5^\circ$$

Example 3: Three charges

Three point charges lie along the x axis. The positive charge $q_1 = 15.0\ mC$ is at $x = 2.00\ m$, the positive charge $q_2 = 6.00\ mC$ is at the origin, and the net force acting on q_3 is zero. What is the x coordinate of q_3?



$$\vec{F}_3 = 0 = \vec{F}_{13} + \vec{F}_{23}$$

$$F_{13} = F_{23}$$

$$\frac{\cancel{k}\,|q_1|\cancel{|q_3|}}{(2-x)^2} = \frac{\cancel{k}\,|q_2|\cancel{|q_3|}}{x^2}$$

$$\Rightarrow \quad q_2(2-x)^2 = q_1 x^2 \quad \Rightarrow \quad \sqrt{q_2}\,|2-x| = \sqrt{q_1}\,|x|$$

$$\pm\sqrt{q_2}\,(2-x) = \pm\sqrt{q_1}\,x$$

$$\sqrt{q_2}\,(2-x) = \pm\sqrt{q_1}\,x$$

$$\sqrt{6\cancel{m}C}\,(2-x) = \pm\sqrt{15\cancel{m}C}\,x$$

$$\Rightarrow\quad 2\sqrt{6} = \left(\sqrt{6} \pm \sqrt{15}\right)x$$

$$\Rightarrow\quad x = \frac{2\sqrt{6}}{\sqrt{6} \pm \sqrt{15}} =$$

or

$\boxed{+\ 0.775\,m}$ ✓

$\cancel{-3.44\,m}$

$|x| = 1$

-1 0 1

$\Rightarrow \boxed{x = \pm 1}$

$\pm(\quad) = \pm(\quad)$

$+\bigcirc = +\bigcirc$

$+ \qquad -$ ②

$- \qquad +$

$- \qquad -$ ①

بالتوفيق والنجاح