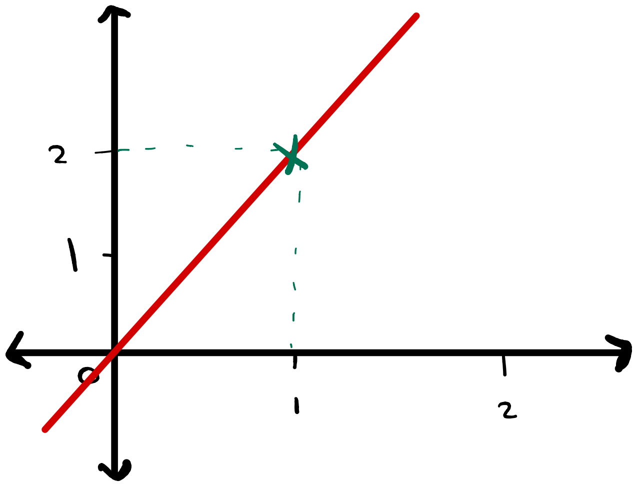
2.2 The Limit of a Function

$$\lim_{x \to a} f(x) = L$$

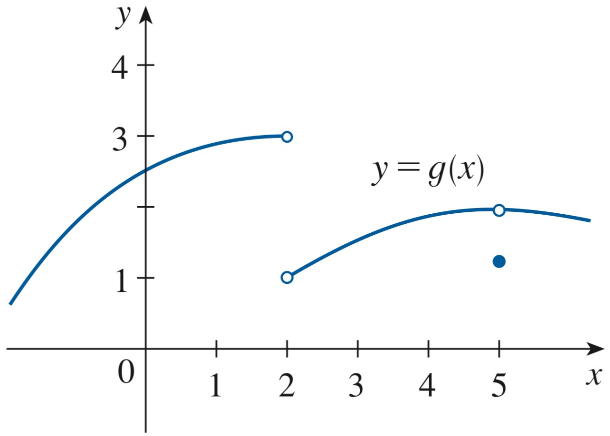
"the limit of $f(x)$, as x approaches a, equals L"

x	$f(x)$
0.9	1.9
0.99	1.99
0.999	1.999
1.001	2.001
1.01	2.01
1.1	2.1



$\lim_{x \to 1^+} f(x) = 2$, $\lim_{x \to 1^-} f(x) = 2 \Rightarrow \lim_{x \to 1} f(x) = 2$

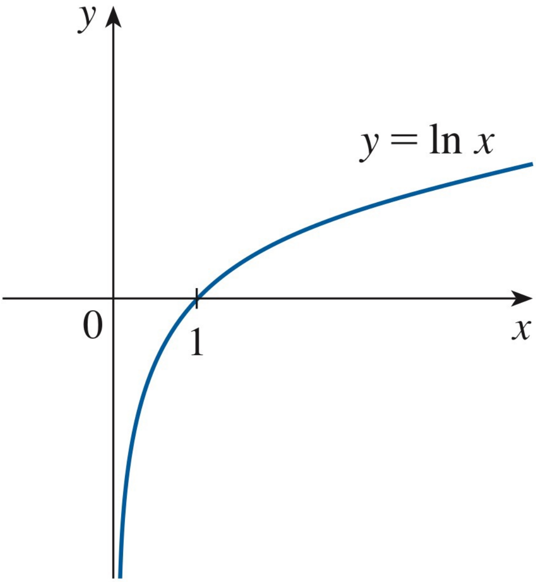
3 $\lim_{x \to a} f(x) = L$ if and only if $\lim_{x \to a^-} f(x) = L$ and $\lim_{x \to a^+} f(x) = L$



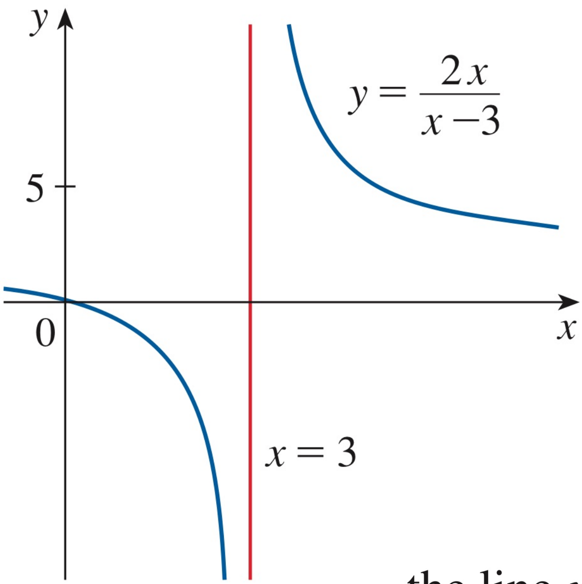
Use the graph to state the values (if they exist) of the following:

(a) $\lim_{x \to 2^-} g(x)$ (b) $\lim_{x \to 2^+} g(x)$ (c) $\lim_{x \to 2} g(x)$

(d) $\lim_{x \to 5^-} g(x)$ (e) $\lim_{x \to 5^+} g(x)$ (f) $\lim_{x \to 5} g(x)$



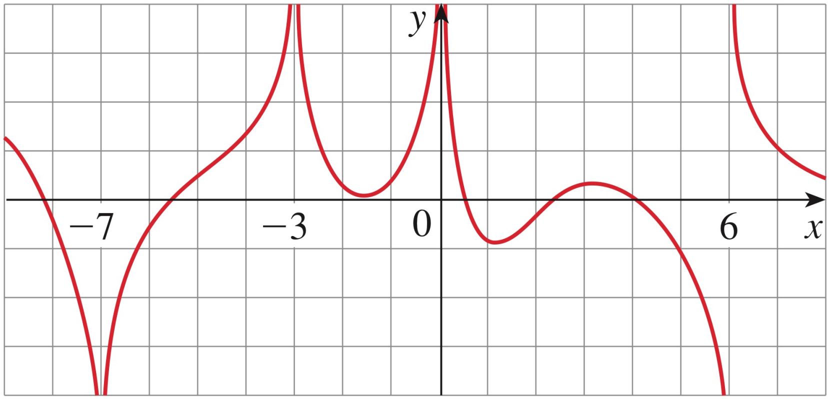
$$\lim_{x \to 0^+} \ln x = -\infty$$



$$\lim_{x \to 3^+} \frac{2x}{x-3} = \infty$$

$$\lim_{x \to 3^-} \frac{2x}{x-3} = -\infty$$

the line $x = 3$ is a vertical asymptote.



9. For the function f whose graph is shown, state the following.

(a) $\lim_{x \to -7} f(x)$

(b) $\lim_{x \to -3} f(x)$

(c) $\lim_{x \to 0} f(x)$

(d) $\lim_{x \to 6^-} f(x)$

(e) $\lim_{x \to 6^+} f(x)$

(f) The equations of the vertical asymptotes

2.3 Calculating Limits Using the Limit Laws

Limit Laws Suppose that c is a constant and the limits

$$\lim_{x \to a} f(x) \qquad \text{and} \qquad \lim_{x \to a} g(x)$$

exist. Then

1. $\displaystyle \lim_{x \to a} [f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$

2. $\displaystyle \lim_{x \to a} [f(x) - g(x)] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x)$

3. $\displaystyle \lim_{x \to a} [cf(x)] = c \lim_{x \to a} f(x)$

4. $\displaystyle \lim_{x \to a} [f(x)\, g(x)] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$

5. $\displaystyle \lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$ if $\displaystyle \lim_{x \to a} g(x) \neq 0$

6. $\displaystyle \lim_{x \to a} [f(x)]^n = \left[\lim_{x \to a} f(x)\right]^n$ where n is a positive integer

7. $\displaystyle \lim_{x \to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to a} f(x)}$ where n is a positive integer

[If n is even, we assume that $\displaystyle \lim_{x \to a} f(x) > 0$.]

8. $\displaystyle \lim_{x \to a} c = c$

9. $\displaystyle \lim_{x \to a} x = a$

10. $\displaystyle \lim_{x \to a} x^n = a^n$ where n is a positive integer

11. $\displaystyle \lim_{x \to a} \sqrt[n]{x} = \sqrt[n]{a}$ where n is a positive integer

(If n is even, we assume that $a > 0$.)

EXAMPLE 1 Use the Limit Laws and the graphs of f and g in Figure 1 to evaluate the following limits, if they exist.

(a) $\lim_{x \to -2} [f(x) + 5g(x)]$

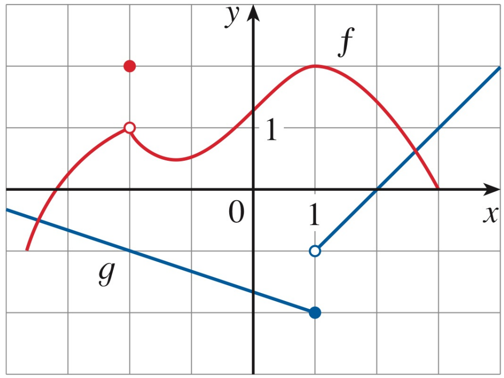


FIGURE 1

(b) $\lim_{x \to 1} [f(x)g(x)]$

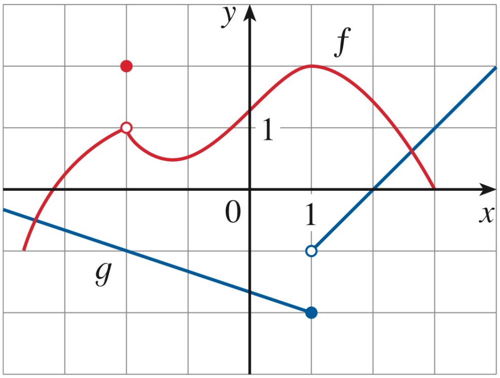


FIGURE 1

(c) $\lim_{x \to 2} \frac{f(x)}{g(x)}$

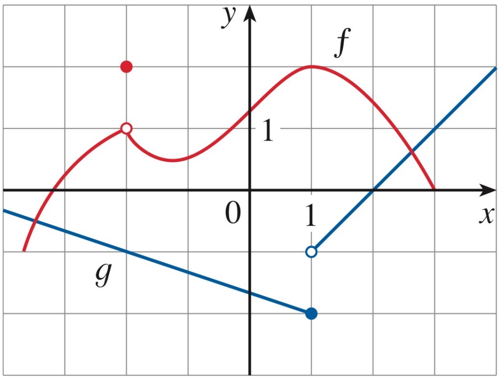


FIGURE 1

Evaluate the limit, if it exists.

11. $\lim_{x \to -2} (3x - 7)$

13. $\lim_{t \to 4} \frac{t^2 - 2t - 8}{t - 4}$

19. $\lim_{t \to 3} \frac{t^3 - 27}{t^2 - 9}$

22. $\lim\limits_{x \to 9} \dfrac{9 - x}{3 - \sqrt{x}}$

28. $\lim\limits_{t \to 0} \left(\dfrac{1}{t} - \dfrac{1}{t^2 + t} \right)$

33. $\lim\limits_{h \to 0} \dfrac{(x + h)^3 - x^3}{h}$

The Squeeze Theorem

3 The Squeeze Theorem If $f(x) \leqslant g(x) \leqslant h(x)$ when x is near a (except possibly at a) and

$$\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L$$

then

$$\lim_{x \to a} g(x) = L$$

39. If $4x - 9 \leqslant f(x) \leqslant x^2 - 4x + 7$ for $x \geqslant 0$, find $\lim_{x \to 4} f(x)$.