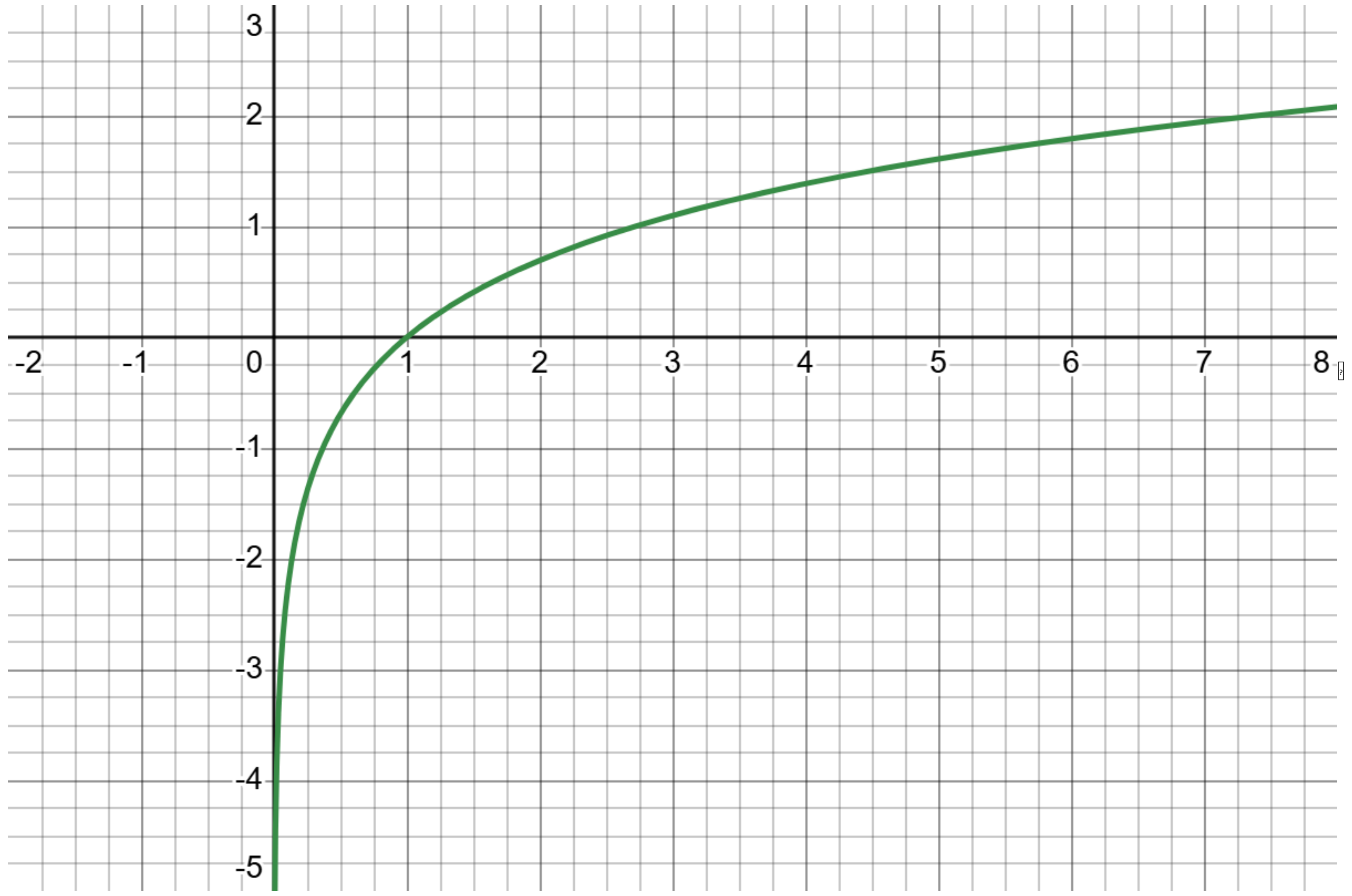
OL Academy

# MATHS102

Lesson 1

## 4.4 L'Hopitals Rule

# Notice that,

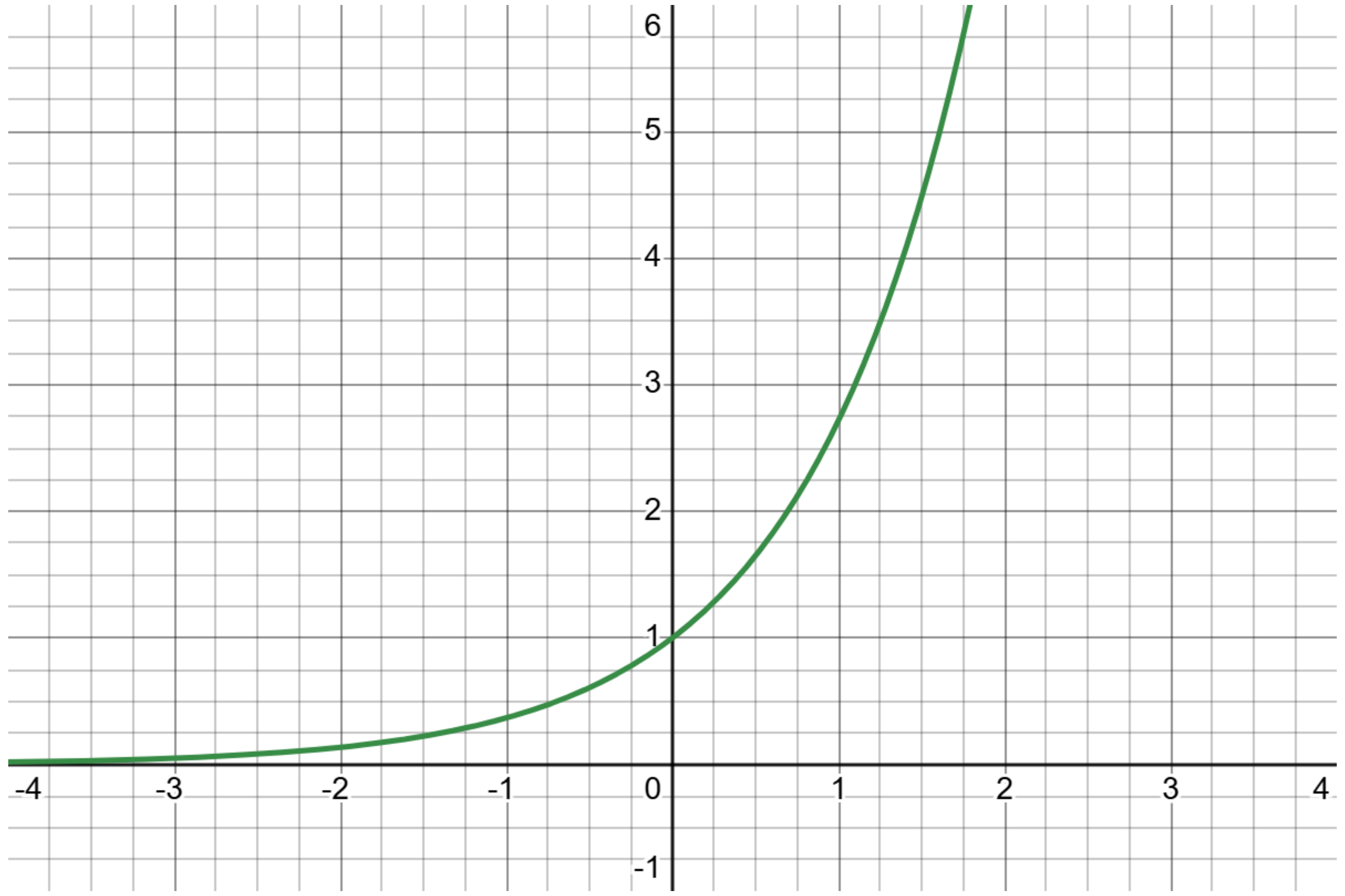



$f(x) = \ln(x)$

Domain $(0, \infty)$

$$\lim_{x \to \infty} \ln(x) = \infty$$

$$\lim_{x \to 0^+} \ln(x) = -\infty$$

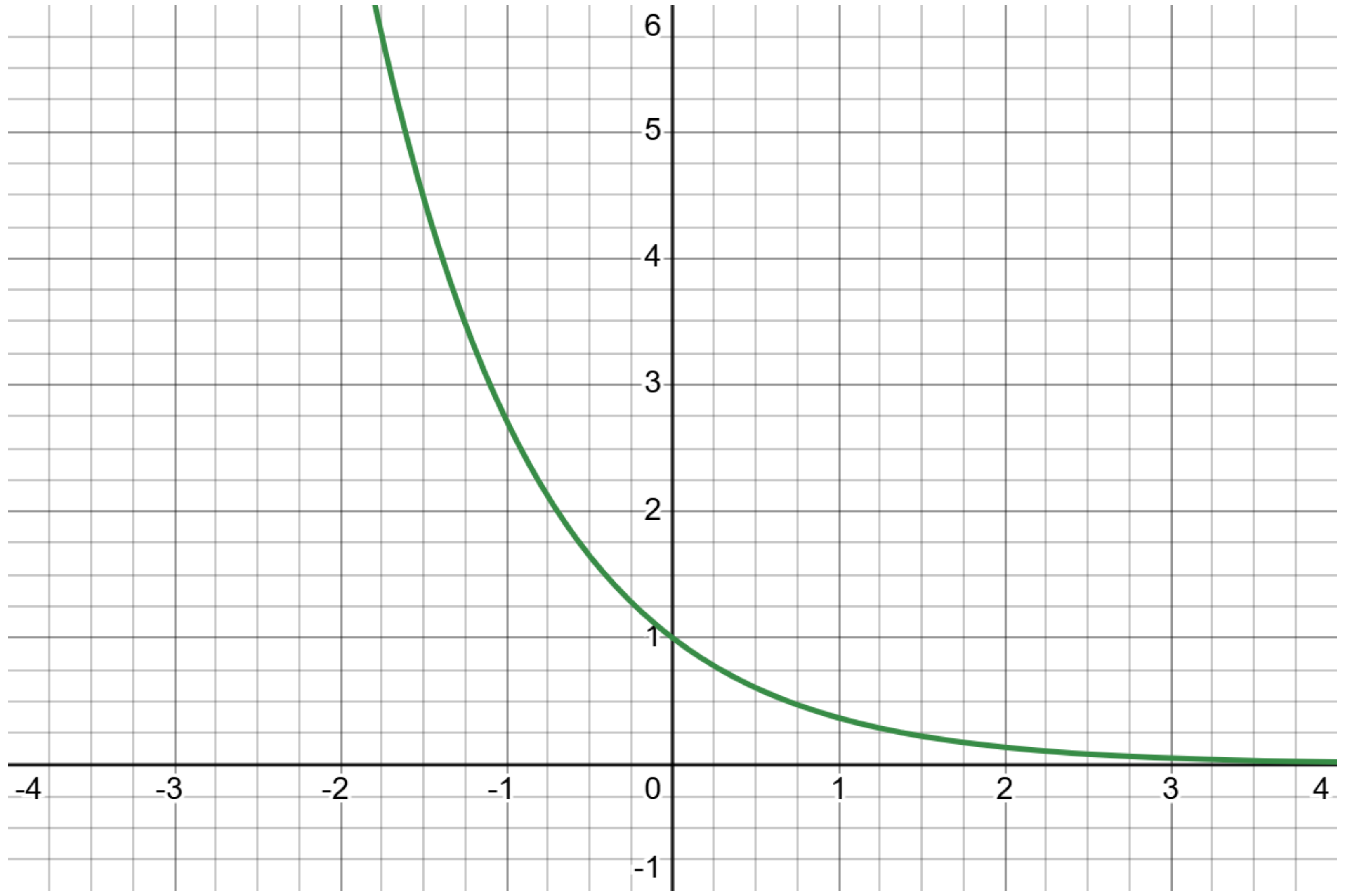



$f(x) = e^x$

Domain $(-\infty, \infty]$

$$\lim_{x \to -\infty} e^x = 0$$

$$\lim_{x \to \infty} e^x = \infty$$



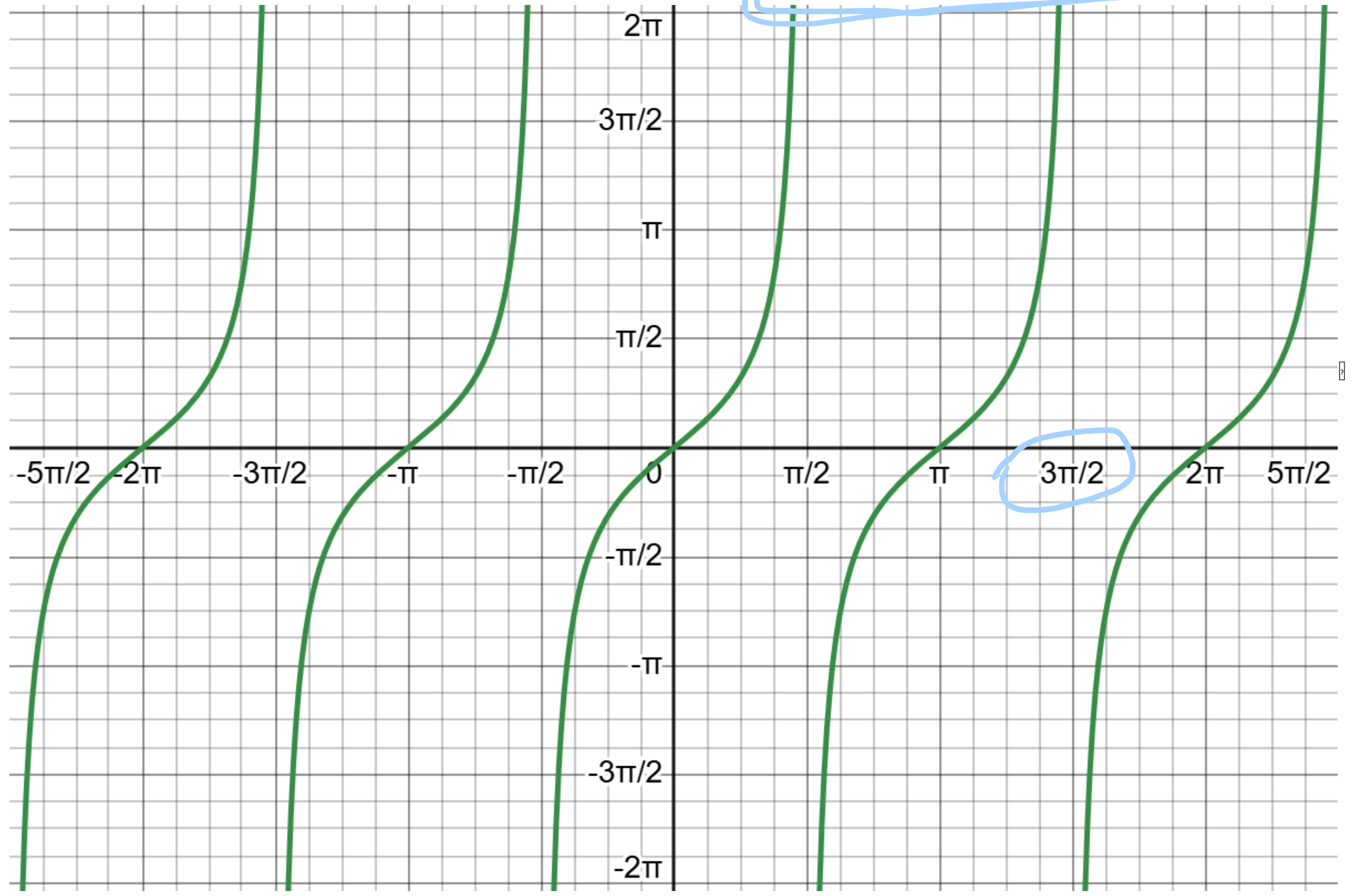

$f(x) = e^{-x}$

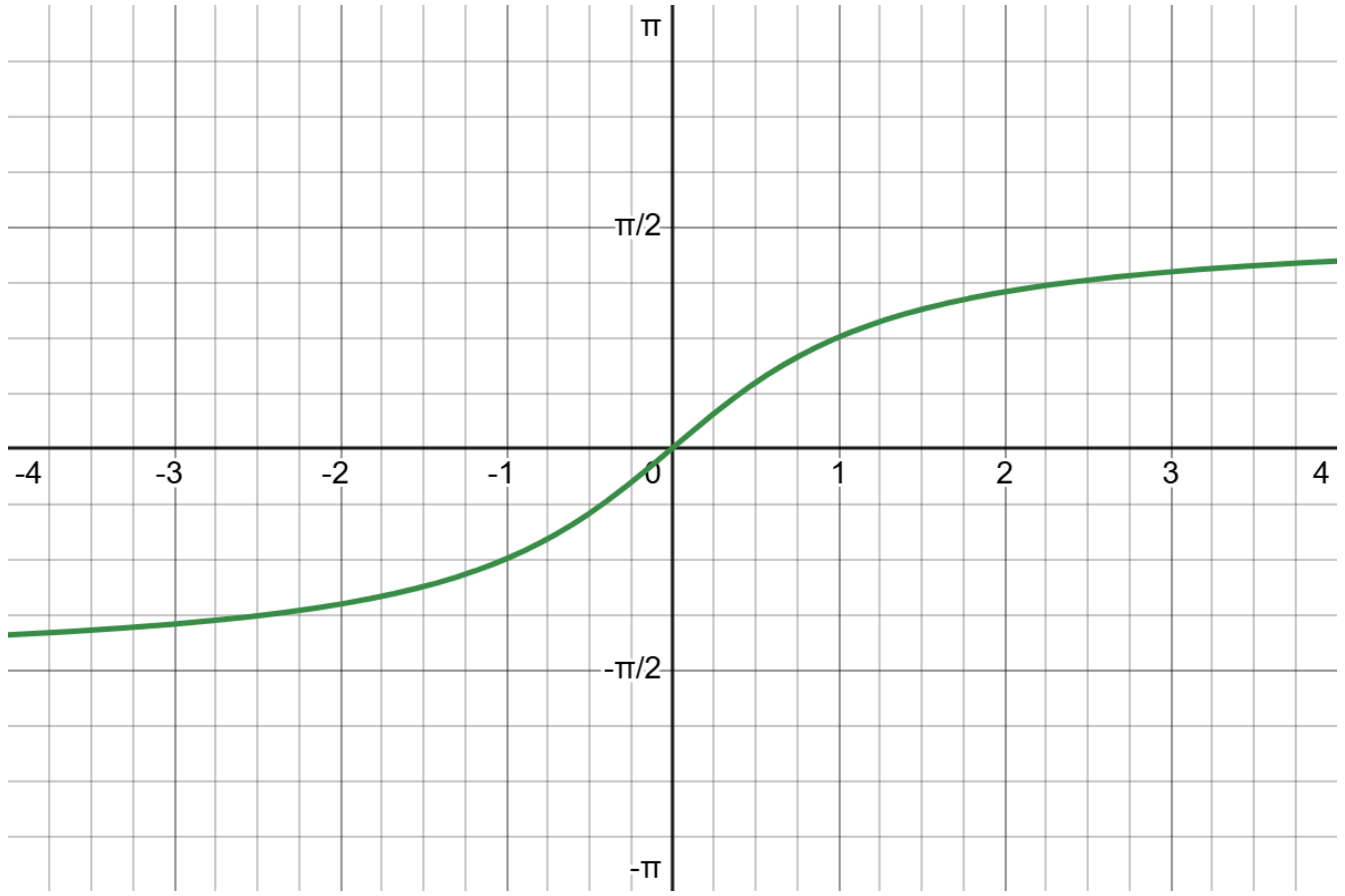
Domain $(-\infty, \infty)$

$$\lim_{x \to -\infty} e^{-x} = \infty$$

$$\lim_{x \to \infty} e^{-x} = 0$$

$\left(x \neq \boxed{\frac{(2n+1)\pi}{2}}, n \in \mathbb{Z}\right)$




$y = \tan(x)$

$$\lim_{x \to \frac{\pi}{2}^+} \tan(x) = -\infty$$

$$\lim_{x \to \frac{\pi}{2}^-} \tan(x) = \infty$$

$f(x) = \tan^{-1}(x)$

$= \arctan(x)$

Domain: $\mathbb{R}$

$$\lim_{x \to \infty} f(x) = \pi/2$$

$$\lim_{x \to -\infty} f(x) = -\pi/2$$

$\frac{1}{0} = \infty$

$\frac{1}{\infty} = 0$

$$a^{\infty} = \begin{cases} \infty, & a > 1 \\ 0, & 0 < a < 1 \end{cases}$$

positive number

$4^{\infty} = \infty$

$(0.5)^{\infty} = 0$

$e^{\infty} = \infty$

2.7

(1) Type $\frac{\infty}{\infty}$, $\frac{0}{0}$ :

$$\lim_{x\to a} \frac{f(x)}{g(x)} \to \frac{0}{0}, \quad \frac{\infty}{\infty}$$

$a \in \mathbb{R}$, $a = \infty$

→ L'Hospital's Rule :

$$\lim_{x\to a} \frac{f'(x)}{g'(x)} = \boxed{\phantom{x}}$$

→ $\frac{0}{0}$, $\frac{\infty}{\infty}$

- $$\lim_{x\to 0} \frac{\sin(x)}{x} \to \lim_{x\to 0} \sin(x) = \sin(0) = 0$$

$$\lim_{x\to 0} x = 0$$

$\frac{0}{0} \Rightarrow$ L'Hospital's Rule.

$$\lim_{x\to 0} \frac{\cos(x)}{1} = \frac{\cos(0)}{1} = 1$$

- $$\lim_{x\to\infty} \frac{e^x}{x^2} \to \frac{\infty}{\infty}$$

$n \neq 0$

$\infty^n = \infty$

$\frac{\infty}{n} = \infty$

$n \cdot \infty = \infty$

$n + \infty = \infty$

$$\lim_{x\to\infty} \frac{e^x}{2x} \to \frac{\infty}{\infty}$$

$$= \lim_{x\to\infty} \frac{e^x}{2} = \frac{e^\infty}{2} = \frac{\infty}{2} = \infty$$

- $\displaystyle \lim_{x \to \frac{\pi}{2}^-} \frac{\sec(x)}{1+\tan(x)} \Rightarrow \frac{\infty}{\infty}$

$$\lim_{x \to \frac{\pi}{2}^-} \frac{\cancel{\sec(x)}\tan(x)}{\sec^{\cancel{2}}(x)}$$

$$\lim_{x \to \frac{\pi}{2}^-} \frac{\tan(x)}{\sec(x)}$$

$$\lim_{x \to \frac{\pi}{2}^-} \frac{\sin(x)}{\cancel{\cos(x)}} \cdot \cancel{\cos(x)}$$

$$\lim_{x \to \frac{\pi}{2}^-} \sin(x) = \sin\left(\frac{\pi}{2}\right) = 1$$

- $\displaystyle \lim_{x \to \infty} \frac{e^{-x}}{\pi/2 - \tan^{-1}(x)} \Rightarrow \frac{0}{0}$

$$= \lim_{x \to \infty} \frac{\cancel{-}e^{-x}}{\cancel{-}\frac{1}{1+x^2}}$$

$$= \lim_{x \to \infty} \frac{(1+x^2)}{e^x} \Rightarrow \frac{\infty}{\infty}$$

$$= \lim_{x \to \infty} \frac{2x}{e^x} \Rightarrow \frac{\infty}{\infty}$$

$$= \lim_{x \to \infty} \frac{2}{e^x} = \frac{2}{\infty} = 0$$