OL Academy

ITCS255

Discrete Structures II

Chapter 1- The growth of function

$$1 < \log n < \sqrt{n} < n < n\log n < n^2 < n^3 \ldots$$

$$2^n < 3^n < n^n$$

3 main notations:

1- Big O $\rightarrow$

2- Big Ω $\rightarrow$ Big Omega

3- Big Θ $\rightarrow$ Big theta

1- **Big O** $\rightarrow$ upper bound

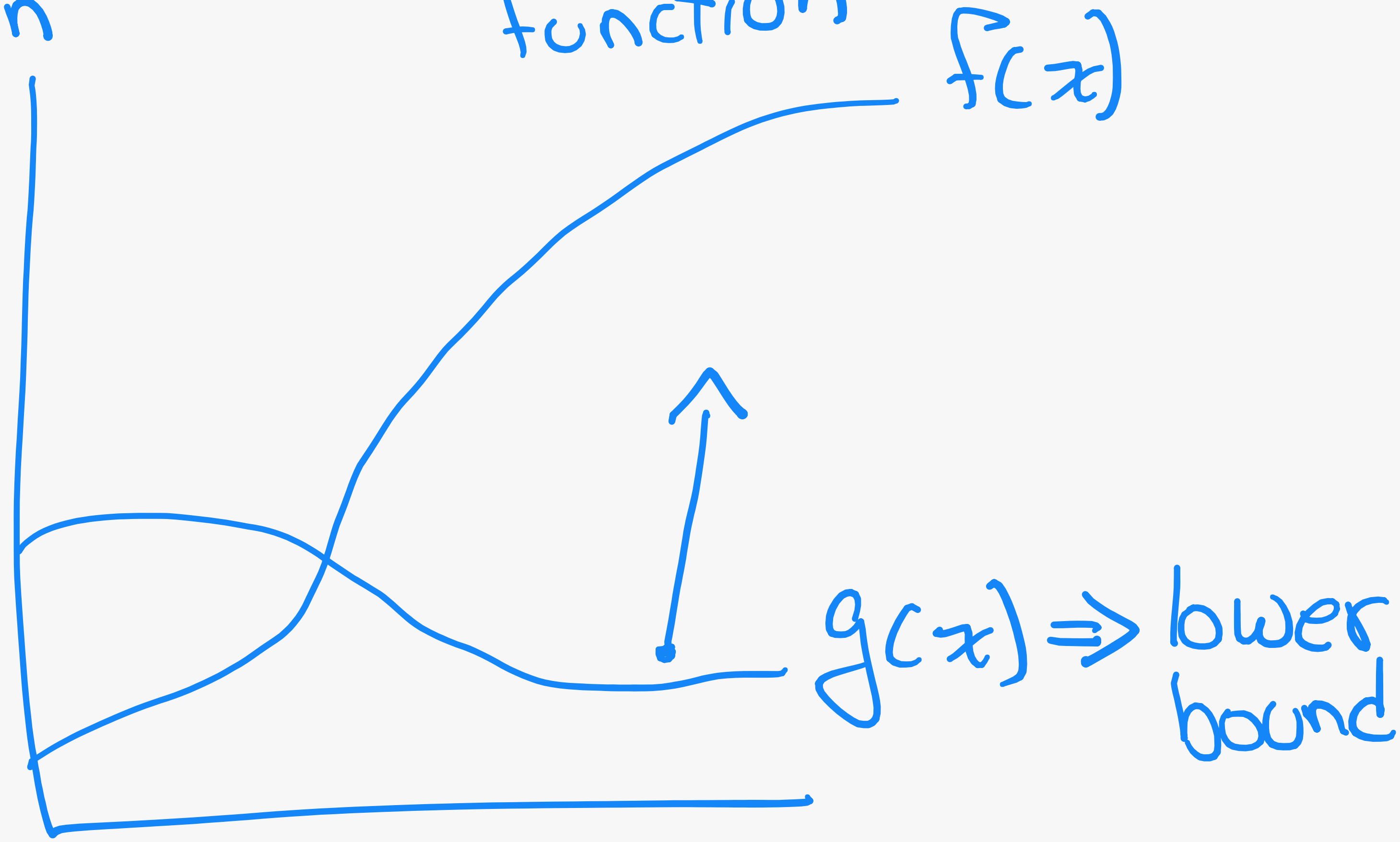
$$\underbrace{f(x)}_{\text{function}} \in O(\underbrace{g(x)}_{\text{function}})$$

$g(x) \rightarrow$ upper bound

$f(x)$

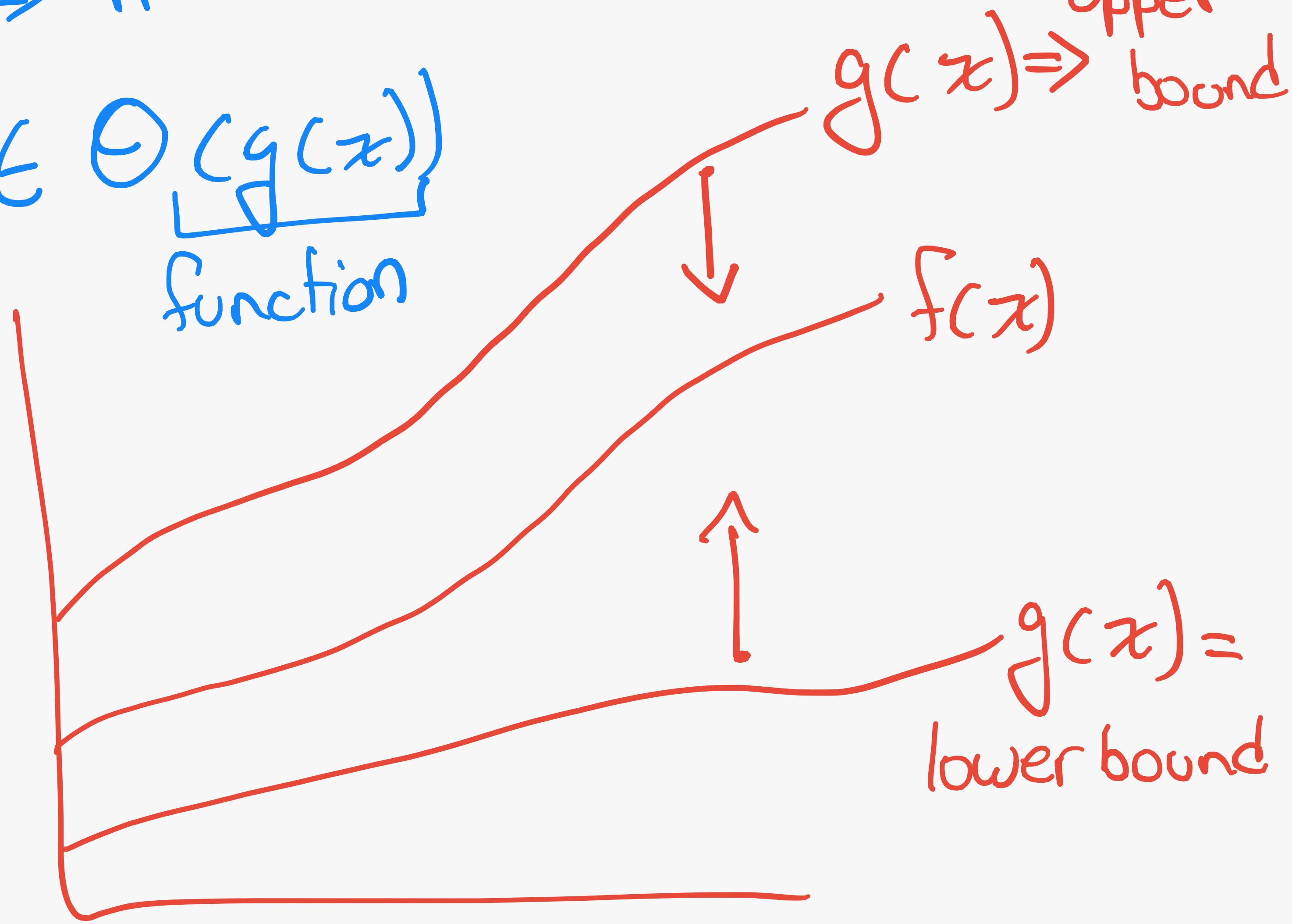
2- Big Ω → lower bound

$$\underbrace{f(x)}_{\text{function}} \in \Omega(\underbrace{g(x)}_{\text{function}})$$



3- Big Θ $\Rightarrow$ upper and lower bound

$$\underbrace{f(x)}_{\text{function}} \in \Theta(\underbrace{g(x)}_{\text{function}})$$



Big O Notation:

Consists of 4 main parts

1- $f(x)$
2- $g(x)$

→ function

3- C
4- k

→ constants

$\Rightarrow |f(x)| \leq C\,|g(x)|$ for all $x > k$

2 Methods to solve Big O

1- Ad-hoc Method $\Rightarrow$ Basic definition

2- General

$$4\boxed{x^5}\ \boxed{-}\ 6x^4\ \boxed{-}\ 2x^2\ \boxed{+}\ 1$$ Find $O(g(x))$

1- We will define the sign of numbers

2- We will define dominant term

dominant term $= x^5$

3- Switch all the negative signs to positive

$$4x^5 - 6x^4 - 2x^2 + 1 \leq 4x^5 + 6x^5 + 2x^5 + 1x^5$$

$$\leq 13x^5$$

$g(x) = x^5$ $\quad C = 13$ $\quad k \geq 1$

$$f(x) = \frac{n^2 + \log n}{n+1}$$ prove that $f(x) \in O(\underbrace{n}_{g(x)})$

$c = ?$

$k = ?$

$g(x) = n$

$$\frac{n^2 + \log n}{n+1}$$

$n^2 + \log n \Rightarrow$ dominant term $= n^2$

$n + 1 \Rightarrow$ dominant term $= n$

$$\frac{n^2+\log n}{n+1} \le \frac{n^2 + n^2}{n}$$

$$\le \frac{2n^2}{n}$$

$$\le 2n$$

$g(x) = n$

$C = 2$

$k \ge 1$