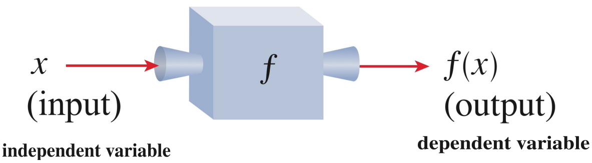
Functions

A **function** f is a rule that assigns to each element x in a set D exactly one element, called $f(x)$, in a set E.

$f(x) = x^2 + 4$ $\qquad$ $x = 0, 2, -2$



$x = 0 \rightarrow f(0) = (0)^2 + 4$

$f(0) = 4$

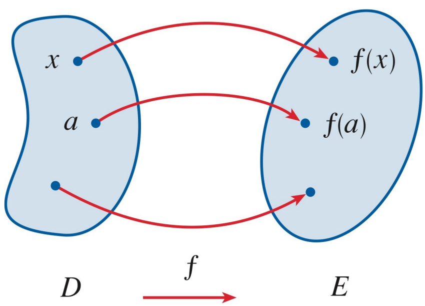
$x = 2 \rightarrow f(2) = (2)^2 + 4$

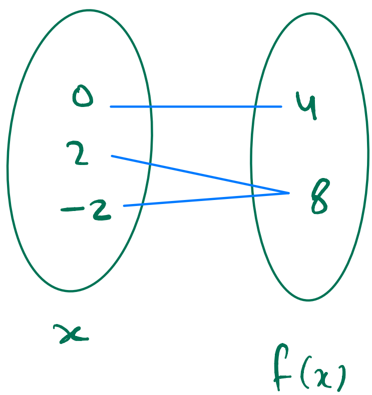
$f(2) = 8$

$x = -2 \rightarrow f(-2) = (-2)^2 + 4$

$4 + 4$

$f(-2) = 8$





Constant Function: $f(x) = C$

Linear Function: $f(x) = aX + b$

Find the value of x :

a) $7 - 4x = 3$

$7 - 3 = 4x$

$4 = 4x$

$x = 1$

b) $$\frac{6 + 3(x-5)}{2} = \frac{4x}{3}$$

$3(6 + 3(x-5)) = 2(4x)$

$18 + 3(3x - 15) = 8x$

$18 + 9x - 45 = 8x$

$9x - 8x = 45 - 18$

$x = 27$

Algebraic Formulas

$(a-b)^2 = a^2 - 2ab + b^2$
$(a+b)^2 = a^2 + 2ab + b^2$
$a^2 - b^2 = (a+b)(a-b)$
$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
$a^3 + b^3 = (a-b)(a^2 - ab + b^2)$
$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$

a) $(2x+3)^2$

$(2x)^2 + 2(2x)(3) + (3)^2 = 4x^2 + 12x + 9$

b) (x^2-5)

$(x^2)^2 - 2(x^2)(5) + (5)^2 = x^4 - 10x^2 + 25$

c) $x^2 - 9$

$(x-3)(x+3) = (x^2 + \cancel{3x} - \cancel{3x} - 9) = x^2 - 9$

d) $x^4 - 16$

$(x^2-4)(x^2+4) \Longrightarrow (x-2)(x+2)(x^2+4)$

Quadratic equation:

$$ax^2 + bx + c = 0$$

Solve:

a) $x^2 - 3x = -2$

$x^2 - 3x + 2 = 0$ mode 5 3

$x = 2$, $x = 1$

$(x-2)(x-1) = 0 \rightarrow x \quad 2 = 0 \rightarrow x = 2$

$x \quad = 0 \rightarrow x = 1$

$(2)^2 - 3(2) + 2 = 4 - 6 + 2 = 0$

$(1)^2 - 3(1) + 2 = 1 - 3 + 2 = 0$

b) $6x^2 - 17x = -12 \rightarrow 6x^2 - 17x + 12 = 0$ mode 5 3 $\frac{3}{2}, \frac{4}{3}$

$a = 6$, $b = -17$, $c = 12$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-17) \pm \sqrt{(-7)^2 - 4(6)(12)}}{2(6)}$$

$$x = \frac{17 \pm \sqrt{289 - 288}}{12} = \frac{17 \pm 1}{12}$$

$x = \frac{18}{12} = \frac{3}{2}$ $x = \frac{16}{12} = \frac{4}{3}$

Logarithmic Function:

$$\log_b x = y \Longleftrightarrow b^y = x$$

Logarithmic properties, If x&y are positive.

1. $\log_b (xy) = \log_b x + \log_b y$

2. $\log_b \left(\frac{x}{y}\right) = \log_b x - \log_b y$

3. $\log_b (x^r) = r \log_b x$

Solve:

a) $\log_2 16 = y$

$\log_b x = y \Leftrightarrow b^y = x$, $\log_2 16 = y \Leftrightarrow 2^y = 16$

$2^4 = 16 \rightarrow 2^4 = 2^4$

$\sqrt[4]{16} = 2$ $\quad y = 4$

b) $\log_2 8 = y$

$\log_b x = y \Leftrightarrow b^y = x$, $\log_3 8 = y \Leftrightarrow 2^y = 8$

$2^3 = 2^3 \rightarrow 2^3 = 8$

$\sqrt[3]{8} = 2$ $\quad y = 3$

Natural Logarithm:

The logarithm with base e => ln

$e \approx 2.718 \ldots$

$\log_e x = \ln x$

$e^{\ln x} = x \quad , \quad e^{\log_e x} = x$

Find the value of x :

a) $e^{3-4x} = 12$

$\cancel{\ln}\, \cancel{e}^{3-4x} = \ln(12)$

$3 - 4x = \ln(12)$

$-4x = \ln(12) - 3$

$x = \dfrac{\ln(12) - 3}{-4} \Rightarrow x = \dfrac{-\ln(12) + 3}{4}$

b) $e^{e^{2x}} = 3$

$\cancel{\ln} \cancel{e}^{e^{2x}} = \ln(3)$

$e^{2x} = \ln(3) \longrightarrow \cancel{\ln} \cancel{e}^{2x} = \ln(\ln(3))$

$2x = \ln(\ln(3))$

$x = \dfrac{\ln(\ln(3))}{2}$