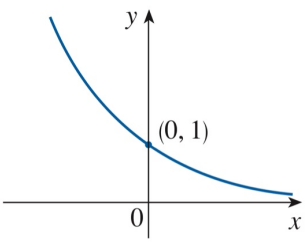
1.4 Exponential Functions

The function $f(x) = 2^x$ is called an *exponential function* because the variable, x, is the exponent.

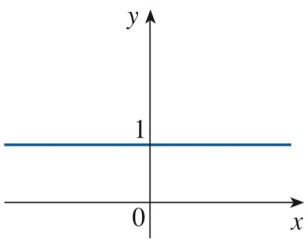
In general, an **exponential function** is a function of the form

$$f(x) = b^x$$

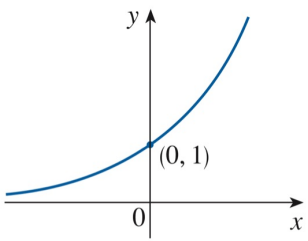
where b is a positive constant.



(a) $y = b^x,\ 0 < b < 1$



(b) $y = 1^x$



(c) $y = b^x,\ b > 1$

Laws of Exponents If a and b are positive numbers and x and y are any real numbers, then

1. $b^{x+y} = b^x b^y$ **2.** $b^{x-y} = \dfrac{b^x}{b^y}$ **3.** $(b^x)^y = b^{xy}$ **4.** $(ab)^x = a^x b^x$

Solving exponential equations:

a) $3^{x+2} = 9^{2x-3}$

b) $8^{4x-12} = 16^{5x-3}$

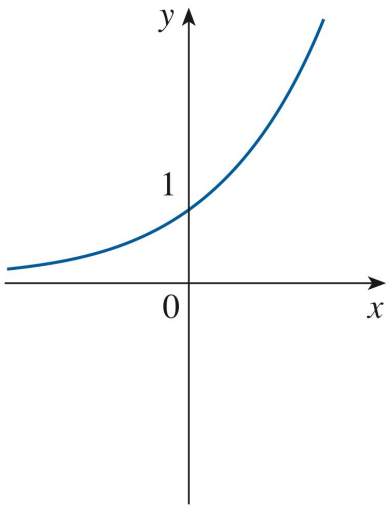
c) $3^{x} = 5$

d) $e^{x} = 5$

e) $2^{x^2} \cdot 4^{x} = 8$

Exponential Function graphing:

$$y = 2^x$$



Domain is always $(-\infty, \infty)$

Range is $(0, \infty)$

$$y = \pm N^{x \pm b} \pm a$$

+N : Draw the graph above the horizontal asymptote.

-N : Draw the graph below the horizontal asymptote.

+b : Move the graph to the left.

-b : Move the graph to the right.

+a : Graph moves up.

-a : Graph moves down.

Examples for graphing:

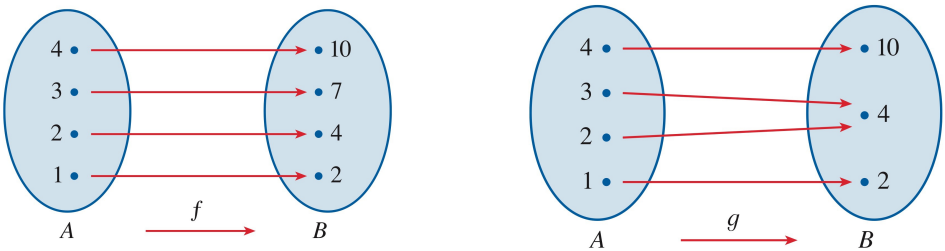
a) $y = 3^{x+1} - 2$

b) $y = 3 - e^{x-2}$

1.5 Inverse Functions and Logarithms

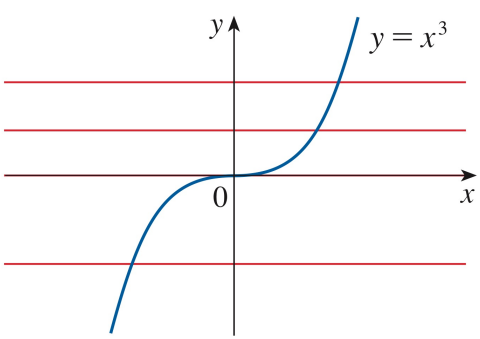
1 Definition A function f is called a **one-to-one function** if it never takes on the same value twice; that is,

$$f(x_1) \neq f(x_2) \qquad \text{whenever } x_1 \neq x_2$$

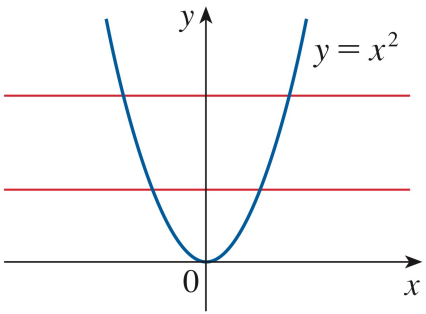


Horizontal Line Test A function is one-to-one if and only if no horizontal line intersects its graph more than once.

EXAMPLE 1 Is the function $f(x) = x^3$ one-to-one?



EXAMPLE 2 Is the function $g(x) = x^2$ one-to-one?



5 How to Find the Inverse Function of a One-to-One Function f

STEP 1 Write $y = f(x)$.

STEP 2 Solve this equation for x in terms of y (if possible).

STEP 3 To express f^{-1} as a function of x, interchange x and y. The resulting equation is $y = f^{-1}(x)$.

Find the inverse of the following:

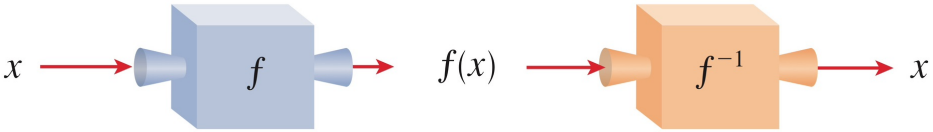
a) $f(x) = 3x + 5$

b) $f(x) = \dfrac{3x - 7}{4x + 3}$

c) $f(x) = \sqrt{2x - 6}$

$$\text{domain of } f^{-1} = \text{range of } f$$

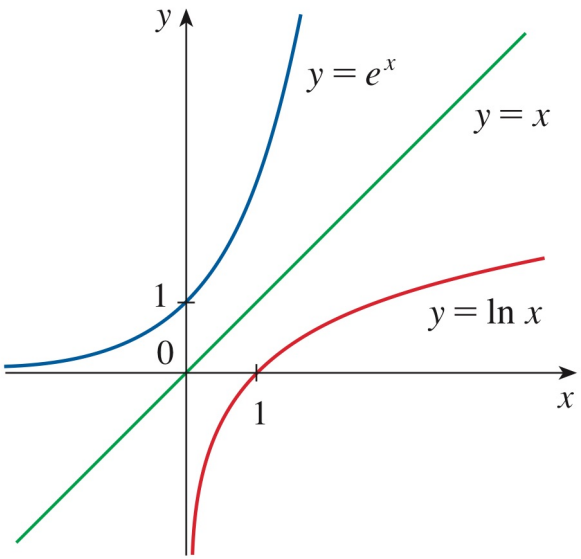
$$\text{range of } f^{-1} = \text{domain of } f$$



$$f^{-1}(f(x)) = x \quad \text{for every } x \text{ in } A$$
$$f(f^{-1}(x)) = x \quad \text{for every } x \text{ in } B$$

d) $f(x) = \sqrt[3]{2x - 7}$

How does the inverse look like?



$y = e^x \Rightarrow$ Domain $(-\infty, \infty)$, Range $(0, \infty)$

$y = \ln x \Rightarrow$ Domain $(0, \infty)$, Range $(-\infty, \infty)$