OL Academy

ITCS254/258 - Discrete Structures I
Final Exam Revision

1. What is the contrapositive of $p \rightarrow (q \rightarrow r)$

2. What is the negation in English "Ahmed will not be happy unless he gets his degree"

3. What is the converse of "It is hot only if it is sunny" in English

4. What are the values of p, q, r, and s that makes the statement false
$$(p \wedge r \rightarrow \neg q) \rightarrow (q \rightarrow r \vee s)$$

5. Write the following in symbolic using the quantifiers e(x), p(x) and s(x) where the domain is the set of all integers

Let e(x): x is even
p(x): x is prime
s(x): x is a perfect square

a. Some integers that are not primes are not perfect squares

b. Any perfect square is not a prime

c. A perfect square number is necessary for being odd

d. No perfect square is even

e. $\forall x: e(x) \rightarrow \neg s(x) \wedge \neg p(x)$

6. Using rules of inference shoe that the following argument is valid

$$p \rightarrow (q \rightarrow r)$$
$$p \vee s$$
$$t \rightarrow q$$
$$\neg s$$
$$\neg r \rightarrow \neg t$$

7. Prove by mathematical induction that $7^n - 2^n$ is divisible by 5 for any integer $n \geq 1$

8. Show that n$(n^2 - 1)(n + 2)$ is divisible by 4 for all integer n

9. Show using set identities that $\underline{A} \cup \underline{B} \cup (A \cap B \cap \underline{C}) = \underline{A} \cup \underline{B} \cup \underline{C}$

10. Answer the following question Given A= {1,2,3}. Let R be relation on A such that R= {(1,1), (2,2), (1,2)} is R reflexive, symmetric, transitive

11. Let U= $\{x \in Z^{+} | \, x \leq 6\}$. A= $\{\in U | \, x \, is \, divisible \, by \, 3\}$.
B= {1} and C= $\{\in U | \, x^2 + 2 = 0\}$

a. $(A \cup C) - B$

b. $\underline{(B \cup C)} - \underline{A}$

c. $P(B \cup C)$

12. Prove by contrapositive if $n^2 + 6n + 5$ is even then n is odd

13. Let f: $R \rightarrow R$ and f(x) = 2x -7 find out is it one to one or onto

14. Draw Venn Diagram for $\underline{A} - (C - B)$

15. Suppose M is the adjacency matrix for an undirected graph G.

$$M = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \end{array}$$

a. Draw the graph

b. Is there a loop in the graph? If yes why?

c. Write the degree of the graph

d. What is the degree of a?

16. Use homogenous technique to solve the following recurrence relation

$$a_0 = 1, a_n = 5a_{n-1} + 4, n > 1$$

17. Suppose M is the adjacency matrix for an undirected graph G.

$$M = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

a. Draw the graph

b. Find the incident matrix for the graph

c. Is the graph bipartite? Why?