ITCS255
Test 2 Revision

1. Please choose the best correct answer for each of the following questions.

   1. The recurrence relation $a_n = a_{n-1} + 4a_{n-2} + 9$ is correct?
      a. does not have constant coefficients.
      b. is nonhomogeneous.
      c. is nonlinear.
      d. is homogeneous.

   2. Which of the following recurrence relations is of order 3?
      a. $5a_n - 4a_{n-2} = 1$
      b. $a_{n+5} + 3a_{n+8} = 3_n$
      c. $a_{n+2} - 4a_n = 1$
      d. $a_{n-5} + 3a_{n-4} = 4n$

   3. Consider the divide-and-conquer recurrence relationT(n) = 2T(n/2) +n. The characteristicpolynomial forT(n) is
      a. (x−2)(x−1)
      b. (x−2)(x−2)
      c. (x−2)$(x-1)^2$
      d. (x−4)(x−2)

   4. A graph is simple if it is connected
      a. True
      b. False

5. The graph with the degree sequence 2,3,3,3 does not exist
   a. True
   b. False

6.

$$M = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \end{array}$$

   a. Draw the graph

   b. Is there a loop in the graph? If yes why?

   c. Write the degree of the graph

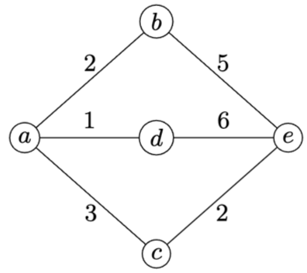
   d. What is the degree of a

7. Use characteristic polynomials to find a solution to the following recurrence relation

$$a_0 = 1,\ a_n = \frac{a_{n-1}}{2^n a_{n-1} + 7}, \quad n \geq 1$$

8. Consider the recurrence relation $a_n = 9a_{n-1} + (4n+2)3^n$. The particular solution toto this recurrence relation is (A+Bn) 3^n. Find the values of A and B. Show your steps

9. Find the shortest path fromatoeusingDijkstra'salgorithm. Please fill thetable below for each step of the algorithm as explained in the class. Also, show your workon the graph itself



10. Find the characteristic equation for the below recurrence relations and find their general solutions without finding the constants

a. $t_n = 3t_{n-1} + 3^n$, $n \geq 1$

b. $T(n) = 6T(\frac{n}{5}) + n$

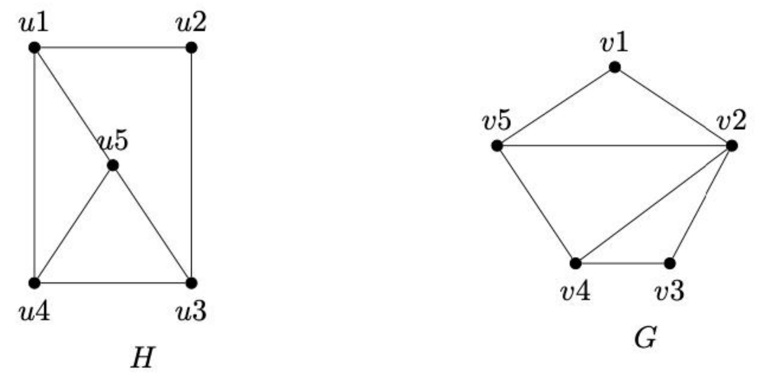
11. Suppose a graphGhasnvertices andeedges. If half of the vertices are all of degree 4 and the other half are all of degree 6, prove that the number of vertices must be even.Do not use drawing for your proof

12. Write thecharacteristic equationas a product of its rootsand its general solution foreach of the following recurrence relations.

Recurrence Relation	**Characteristic Equation**	**General Solution**
$a_n = -a_{n-1} + 1$		
$a_n = 7a_{n-1} + 5a_{n-2} - 75a_{n-3}$		
$a_n = a_{n-1} - (3n+1)(-4)^n$		

MORE ON GRAPHS:

1. Determine whether the given pair of graphs is isomorphic. Exhibit an isomorphism or provide a rigorous argument that none exists.



Need to check the 4 condition for the graphs to be isomorphism which are:
1-The number of vertices must be the same
2-The number of edges must be the same
3-The degree sequence must be the same
4-If the cycle is formed by vertices (u1,u2,u3) in graph 1 then another graph should also form a cycle using the vertices (v1,v2,v3)

Step 1:Check the number of vertices in each graph

From the figure it's clear that:
In graph H
The vertices are: u1, u2, u3, u4, u5.
Total number of vertices in graph H = 5

From the figure it's clear that:
In graph G
The vertices are: v1, v2, v3, v4, v5.
Total number of vertices in graph G = 5

Therefore, the number of vertices in both graphs is the same as 5

Step 2: Check the number of edges in each graph

In graph H
The edges are u1,u2 , u1,u4 , u1,u5 , u2,u3 , u3,u4 ,u3,u5 ,u4,u5
The total number of edges in graph H = 7

In graph G
The edges are v1,v2 , v1,v5 , v2,v3 , v2,v4 , v2,v5 ,v3,v4 ,v4,v5
The total number of edges in graph G = 7

Therefore, the number of edges in both graphs is the same as 7

Step 3: Check the degree sequence

In graph H
The degree sequence are u1=3 , u2=2 , u3=3 , u4=3 u5=3

In graph G
The degree sequence are v1=2 , v2=4 , v3=2 , v4=3 , v5=3

By placing the degree sequence for each graph in ascending order
H = 2,3,3,3,3
G = 2,2,3,3,4
Which makes it clear that the two graphs do not have the same degree sequence

Therefore, graph H and graph G are not isomorphism