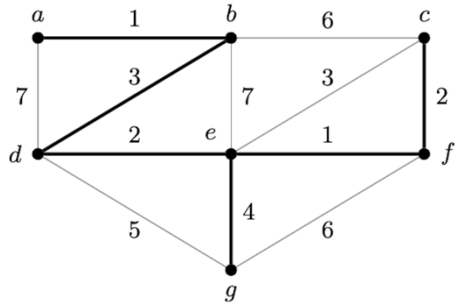
ITCS255
FINAL REVISION

1. Consider the following graph



Find the minimum spanning tree for the graph starting from the vertex d.

2. For the following pairs of function determine which function grows faster using limit theorem.

$$f(x) = x^2 log x, \quad g(x) = x^3$$

3. Let $a_0 = 0, a_1 = 1, a_n = 3a_{n-1} + 10a_{n-2} + 14.5^n$

Find the solution of a_n using the characteristic polynomial of a_n
To find the constants of the general solution, use only boundary condition.

4. Consider the tree with root node 1 and edges
{1,2},{1,3},{1,4},{2,5},{2,6},{3,7}{4,8,},{4,9},{7,10},{7,11}

Find

a. The value of m for m-ary tree
b. Whether or not the tree is full m-ary tree. Justify
c. Whether or not the tree is a complete m-ary tree. Justify
d. The height of the tree

5. A full 6-ary tree with 19 vertices. Find the number of the internal vertices and the number of leaves.

6. Use homogenous technique to solve the following recurrence relation

$$a_0 = 1, a_n = 5a_{n-1} + 4, n > 1$$

7. Suppose M is the adjacency matrix for an *undirected* graph G.

$$M = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

a. Draw the graph

b. Find the incident matrix for the graph

c. Is the graph bipartite? Why?

d. Which edges can you remove to form a spanning tree

e. Draw the spanning tree found in d as a rooted tree with c as it's root

8. For the function f defined by f(n) = $\frac{n^2+1}{n+1}$ for n E N, show that f(n) $\in$ O(n). Use the Ad-hoc Calculations.

9. Find a solution to 13x = 6 (mod 7611)

10.Give a big-O estimate for f(n)= $(nlgn + n^2)(n^3 + 2)$

11.How many strings can be formed by ordering the letters ABCDEFGH subject to the givenconditions.

a. Contains the letters AC together in any order.

b. Contains the letter ABC (together in any order) and DEF (together in any order).

c. Contains either the letters ABC (together in any order) or DEF (together in anyorder).

12. From among a group of 6 different models of TOYOTA cars and 9 different models of GMCcars, 5 cars to be donated as gifts to different charity societies. How many such arrangement ofdonations can be made such that:

a. Any car is eligible for donation

b. Three TOYOTA cars and two GMC cars are to be donated

c. At least three GMC cars are to be donated

13. Find n when $3P(n+1,3) - 4P(n,3) = 0$, where $n \geq 3$