OL Academy

ITCS255
Discrete Structures II

Chapter 4
Combinatorics

66939059

olearninga
www.olearninga.com

Pascal triangle

$k \rightarrow$ / $n \downarrow$	0	1	2	3	4	5
0	1					
1	1	+ 1	+ 0			
2	1	2	+ 1	+ 0		
3	1	3	3	1		
4	1	4	6	+ 4	1	
5	1	5	10	10	5	1

$\longrightarrow$

Expansion $(x+y)^n =$

n	
0	1
1	$x+y$
2	$x^2+2xy+y^2$
3	$x^3+3x^2y+3xy^2+y^3$
4	$x^4+4x^3y+6x^2y^2+4xy^3+y^4$
5	$x^5+5x^4y+10x^3y^2+10x^2y^3+5xy^4+y^5$

Pascal identity =

Table entry:

$$\left(\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k} \right)$$

Expansion:

$$(x+y)^n = \sum_{k=1}^{n} \binom{n}{k} x^{n-k} y^k$$

Binomail Theorem

Pascal identity: $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$

Binomail Theorem: $(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$

number of terms in binomail expantion

$(n+1)$

$k=0$

$(k+1)^{th}$ term $= \binom{n}{k} x^{n-k} y^k$

Find 4^{th} term in the expansion $(2x-\frac{1}{3})^8$

$(k+1)^{th}$

$(3+1) \Longrightarrow k=3$

$$= \binom{8}{3} \cdot 2x^5 \cdot \left(\frac{1}{3}\right)^3$$

$n=8$

$$t_{k+1} = \binom{n}{k} \cdot x^{n-k} \cdot y^k$$

$$= \binom{8}{3} \cdot x^{8-3} \cdot y^3$$

Find the coefficient of $x^5 y^{\underline{\underline{4}}}$ in expansion $(3x+5y)^9$

$$\binom{9}{k}(3x)^{9-k}(5y)^{k}$$

$$=\binom{9}{k}(3)^{9-k}(5)^{k}(x)^{9-k}(y)^{k} \quad k=4$$

$$=\binom{9}{4}(3)^{9-4}(5)^{4}(x)^{9-4}(y)^{4}$$

$$=\binom{9}{4}(3)^{5}\cdot 5^{4} \Longrightarrow 126$$

Find cofficient of x^6 is the expantion of

$$\left(3x^2 - \frac{1}{3x}\right)^9$$

$$\Rightarrow t_{k+1} = \binom{9}{k} (3x^2)^{9-k} \left(\frac{-1}{3x}\right)^k$$

$$= \binom{9}{k} (3)^{9-k} \left((x)^2\right)^{9-k} \left(\frac{-1}{3}\right)^k x^{-k}$$

$$= \binom{9}{k} (3)^{9-k} (x)^{18-2k} \left(\frac{-1}{3}\right)^k x^{-k}$$

$$= \binom{9}{k}(3)^{9-k}\left(-\frac{1}{3}\right)^{k} \cdot x^{18-2k} \cdot x^{-k}$$

$$= \binom{9}{k}(3)^{9-k}\left(-\frac{1}{3}\right)^{k} x^{18-3k}$$

$$= \binom{9}{4}(3)^{9-4}\left(-\frac{1}{3}\right)^{4} x^{18-3(4)}$$

$$= \binom{9}{4}(3)^{5}\left(-\frac{1}{3}\right)^{4} x^{6}$$

$18-3k=6$

$-3k=6-18$

$\frac{-3k}{-3}=\frac{-12}{-3}$

$k=4$

coefficient $=$

$\binom{9}{4}(3)^5\left(-\frac{1}{3}\right)^4$