ITCS255
Test 1 Revision

1. Please choose the best correct answer for each of the following questions.

   1. Suppose thatf(x) is an upper bound function ong(x). Which of the following iscorrect?
      a. $g(x) \in O(f(x))$
      b. $g(x) \in \Omega(f(x))$
      c. $g(x) \in \Theta(f(x))$

   2. Suppose a and b are integers and $a|b$. Which of the following is correct?
      a. $a|(b-5)$
      b. $a|5b$
      c. $b|a$
      d. $\gcd(a, b) = 1$

   3. The linear congruence $a\,x \equiv b \pmod{m}$ has a unique solution if
      a. $\gcd(a, b) = 1$
      b. $\gcd(b, m) = 1$
      c. $\gcd(a, m) = 1$
      d. $\gcd(a, m)|b$

   4. The linear combination of gcd(17,13) is
      a. $17\times(-2) + 13\times5$
      b. $17\times(-1) + 13\times5$
      c. $17\times(-3) + 13\times4$
      d. $17\times(-2) + 13\times4$

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5. Which of the following is true?
   a. 15≡4 (mod 7)
   b. 15≡2 (mod 3)
   c. 21≡3 (mod 6)
   d. 17≡4 (mod 6

6. For the functions, $n^{1000000}$ and 2^n, what is the asymptotic relationship between these func-tions?
   a. $n^{1000000}$ is O(2^n)
   b. $n^{1000000}$ is Ω(2^n)
   c. $n^{1000000}$ is Θ(2^n)

7. Given that 8≡3 (mod 5) and 9≡4 (mod 5), which of the following is true ?
   a. $8^9 \equiv 3^4$ (mod 5)
   b. 9/8 ≡ 4/3 (mod 5)
   c. 8 + 9 ≡ 3 + 4 (mod 5)
   d. 8 ≡ 4 (mod 5) and 9 ≡ 3 (mod 5)

2. Use Arithmetic Modularoperations to find 15^{5630}(mod 14)

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3. Let n and m be integers. Prove that if n | m, then n | $(4n - 5m)^2$

4. Use the Basic Definition of big-Oto prove that n + log_2(n+ 1)∈O(n). Apply the Ad-hoc calculations method.

5. Use infinite limits to prove that $\frac{4n^3+2}{n^2}$ ∈Θ(n)

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6. Consider the linear congruence $55x \equiv 1570 \pmod{22570}$

   a. How many solutions does it have? Simplify it so that it has a unique solution

   b. Find the unique solution to the simplified linear congruence you found in part(a). Show your steps.

7. Letf(n) = $n^2 - 4n + 1$. Use the basic definition to prove that $f(n) \in O(n^2)$.

8. If $k \in N$, prove that $\gcd(3k+2, 5k+3) = 1$.

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9. Suppose a+b ≡ 4 (mod 5) and 3a+b ≡ 12 (mod 5). Find the value of a

10. Solve the linear congruence 19x≡1 (mod 77)

11. Give a big-Oestimate forf(n) = (n lgn+n^2)(n^3+ 2)

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12. UseForward Chainingto solve the following recurrence relation.

$$a_0 = 0, \quad a_0 = a_{n-1} + 3n, \quad n \geq 1$$

13.Use Back Substitution method to solve the recurrence relation

$$a_1 = 3, \ a_n = 7a_{n-1} + 6, \quad n \geq 2$$

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